HYDRAULIC MACHINES
&
INDUSTRIAL FLUID POWER

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Diploma Engineering

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HYDRAULIC TURBINE

Introduction to Hydraulic machines:

- These are the machines in which force is transmitted by means of motion of fluid under pressure.
- These can convert hydraulic energy to mechanical energy or mechanical energy to hydraulic energy.
- The hydraulic system works on the principle of *Pascal's law*. This law states that, the pressure in an enclosed fluid is uniform in all the directions.
- *Examples*: Hydraulic turbines, Pumps, cranes, forklifts, bulldozers

Hydraulic turbine:

- It is a hydraulic machine.
- It uses energy of flowing water (hydraulic energy) and converts it into mechanical energy (in the form of rotation of runner)
- Shaft power available at the shaft of the Turbine is utilized to run Generator to produce electricity.

Classification of turbine:

- *According to the type of energy at inlet*
    - Impulse turbine
        - An impulse turbine is a turbine in which the water entering the runner possesses kinetic energy only. In this, the rotation of the runner occurs due to the impulse action of water. (Pelton Turbine)
    - Reaction turbine
        - A reaction turbine is a turbine in which the water entering the runner possesses pressure as well as kinetic energy. In this, the rotation of runner occurs due to the pressure difference between the inlet and outlet of the runner. (Francis and KaplanTurbine)
- *According to the direction of flow through runner*
    - Tangential flow turbine
        - When the flow of water is tangential to the wheel circle, the turbine is called tangential flow turbine. (Pelton Turbine)
    - Radial flow turbine
        - When the water moves along the vanes towards the axis of rotation of the runner or away from it, the turbine is called radial flow turbine. When the flow is towards the axis of rotation, the turbine is called an inward flow turbine. When the flow is away from the axis of rotation, the turbine is called an outward flow turbine. (Francis Turbine)
    - Axial flow turbine
        - When the water flows parallel to the axis of rotation, the turbine is called an axial or parallel flow turbine. (Kaplan Turbine/Propeller Turbine)

- Mixed flow turbine
  - When the water enters radially inwards at inlet and discharge at outlet in a direction parallel to the axis of rotation of the runner, the turbine is called mixed flow turbine. (Moden Francis Turbine)

- *According to the head at the inlet of turbine*
  - High head turbine
    - When a turbine works under a head of more than 250 m. (Pelton Turbine)
  - Medium head turbine
    - When a turbine works under a head of 45 m – 250 m. (Francis Turbine)
  - Low head turbine
    - When a turbine works under a head of less than 45 m. (Kaplan Turbine)
- *According to the specific speed of the turbine*
  - Low specific speed turbine
    - The specific speed up to 30 (Pelton Turbine)
  - Medium specific speed turbine
    - The specific speed varies from 50 to 250 (Francis Turbine)
  - High specific head turbine
    - specific speed is more than 250 (Kaplan Turbine)

Impulse Turbine – Pelton Wheel:

- Pelton turbine is a *tangential flow* impulse turbine.
- It works at high head and requires low flow of water.
- It converts pressure energy into kinetic energy in in one or more nozzles.
- It is driven by high velocity jets of water coming out from a nozzle directed on to vanes or buckets attached to a wheel.
- The impulse provided by the jets is used to spins the turbine wheel and removes kinetic energy from the fluid flow.
- Pressure of water remains atmospheric inside the turbine.

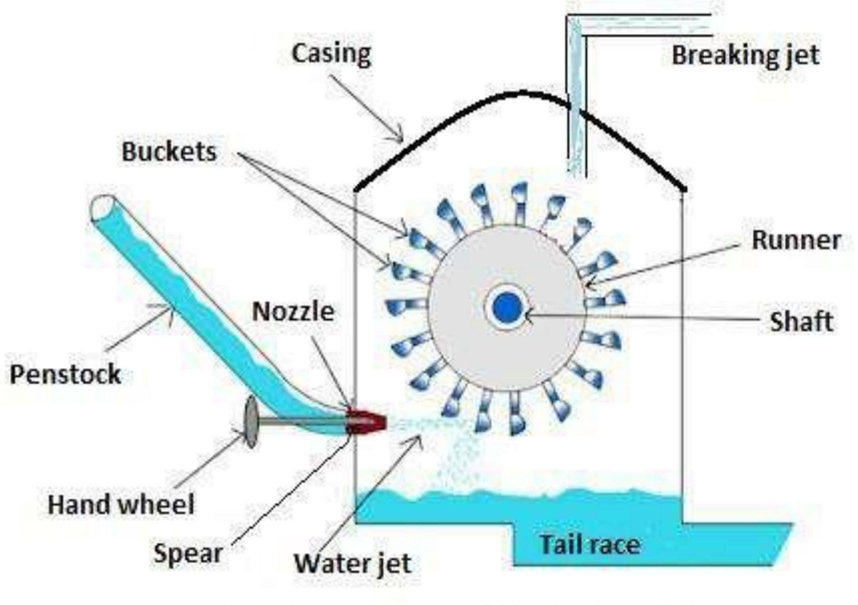
Construction of Pelton Wheel:

Major Components component of Pelton wheel are described below.

- Casing:
  - Casing prevents the splashing of water and helps in discharge of water from the nozzle to the tailrace. It protects the turbine from dust and dirt.
- Nozzle and Spear Mechanism:
  - Nozzle produces high velocity jets of water and converts pressure energy into kinetic energy.

- The spear mechanism controls the water flow into the turbine and control the turbine speed according to load. It minimizes energy loss at inlet and provides smooth flow.

- Break Nozzle:
  - It is used to produce and supply breaking jet of water. It directs the water on the bucket to stop the runner to rest in a short time
- Runner/Rotor:
  - It is a circular disc mounted by a number of equally spaced buckets which are fixed on its periphery. Each bucket consists of two symmetrical halves having shape of semi-ellipsoidal cup.
  - It provides rotational energy when jet of water having kinetic energy strike the buckets.
- Penstock:
  - It is the channel or pipeline that connect the high head source water to the power station
- Governing Mechanism:
  - It controls the speed and power output of the turbine by controlling the flow of water



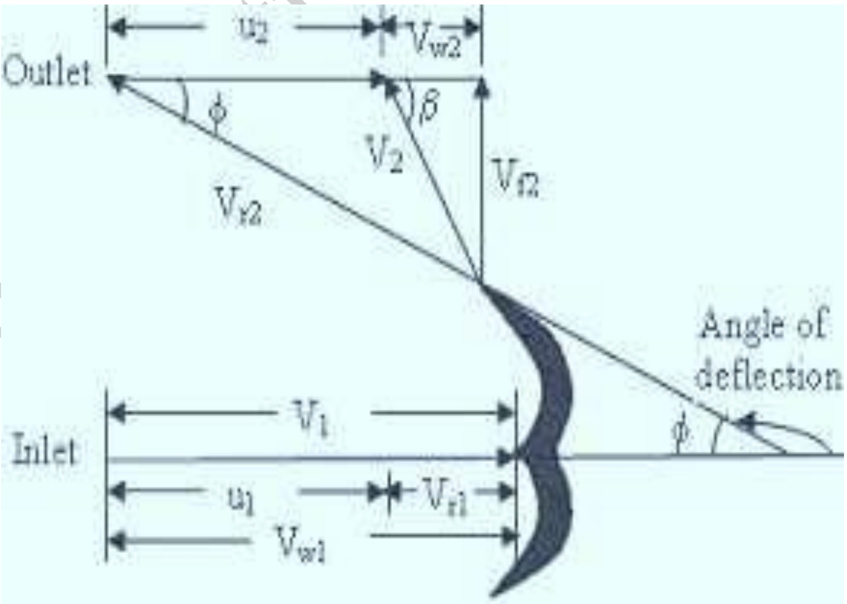
Working Principle:

- Water is coming from the storage reservoir through a penstock to the inlet of the nozzle.
- Nozzle converts the hydraulic energy of the water into kinetic energy and produces high velocity of jet.
- The jet of water released from the nozzle strikes on the buckets mounted on the runner.
- Water jet strikes over the runner bucket and imparts a very high impulsive force on the buckets for a small amount of time to rotate the runner and so mechanical energy develops.
- Pressure of water remains atmospheric inside the turbine.

Velocity triangle of Impulse turbine:

Consider the following terms for understanding the velocity triangle.

At inlet velocity triangle:	*At outlet velocity triangle:*
V_1 = absolute velocity of water	V_2 = absolute velocity of water
u_1 = peripheral velocity of runner (bucket speed)	u_2 = peripheral velocity of runner (bucket speed)
Vr_1 = relative velocity of water	Vr_2 = relative velocity of water
Vw_1 = velocity of whirl	Vw_2 = velocity of whirl
Vf_1 = velocity of flow	Vf_2 = velocity of flow
α = angle between the direction of the jet and the direction of motion of the vane (*guide blade angle*)	β = angle between the direction of the jet and the direction of motion of the vane (*guide blade angle*)
θ = angle made by the relative velocity Vr_1 with the direction of motion (*vane angle*)	φ = angle made by the relative velocity Vr2 with the direction of motion (*vane angle*)
From inlet velocity triangle we obtain:	*From outlet velocity triangle we obtain:*
$\alpha = 0$, $\theta = 0$, $Vf_1 = 0$ $V_1 = Vw_1 = u_1 + Vr_1$ and $Vr_1 = V_1 - u_1$	$Vr_2 = Vr_1$ $Vw_2 = Vr_2 \cos\varphi - u_2$
$u = u_1 = u_2 = \pi DN/60$, where D = diameter of wheel, N = speed in r.p.m	



(Velocity triangle of Pelton turbine)

Work done and power developed of a Pelton wheel:

Let, F = force exerted by the jet of water in the direction of motion

= mass x change in velocity in the direction of force

= m x ($Vw_1 + Vw_2$) = $\rho a V_1$ x ($Vw_1 + Vw_2$)

Where: ρ = density of water;

a = area of jet = $\frac{\pi}{4} \times d^2$

d = diameter of jet

Let, W = Net work done by the jet on runner per second = F x u

= $\rho a V_1$ x ($Vw_1 + Vw_2$) x u

Work done per second per unit weight of water striking = $\frac{\rho a V_1 \times (V_{w_1} + V_{w_2}) \times u}{\rho a V_1 \times g} = \frac{(V_{w_1} + V_{w_2}) \times u}{g}$

NOTE:

Gross head (H_g): - Difference between the water level at head race and tail race

Net head (H): - Head available at the inlet (Effective head)

Absolute velocity can be obtained as: $V_1 = C_V \sqrt{2gH}$

Cv = coefficient of velocity of the nozzle

Velocity of wheel (bucket speed) = $u = \emptyset \times \sqrt{2gH}$

∅ is the speed ratio

Efficiencies of turbine:

1) Hydraulic efficiency (η_h):

$$\eta_h = \frac{Work\ done\ per\ second}{Kinetic\ energy} = \frac{\rho a V_1 \times (Vw_1 \pm Vw_2) \times u}{\frac{1}{2} \times (\rho a V_1) \times V_1^2} = \frac{2 \times (Vw_1 \pm Vw_2) \times u}{V_1^2}$$

It can also be obtained as:

$$\eta_h = \frac{Runner\ Power}{Water\ Power} = \frac{\frac{\rho a V_1 \times (Vw_1 \pm Vw_2) \times u}{1000}\ kW}{\frac{\rho g Q H}{1000}\ kW} = \frac{\rho \times Q \times (Vw_1 \pm Vw_2) \times u}{\rho g Q H}$$

$$= \frac{(Vw_1 \pm Vw_2) \times u}{g H}$$

2) Mechanical efficiency (η_m):

$$\eta_m = \frac{Shaft\ Power}{Runner\ Power} = \frac{P}{\rho a V_1 \times (V w_1 \pm V w_2) \times u}$$

3) Volumetric efficiency (η_m):

$$\eta_v = \frac{volume\ of\ water\ actually\ striking\ the\ runner}{total\ water\ given\ by\ the\ jet\ to\ the\ turbine} = \frac{Q_a}{Q}$$

4) Overall efficiency (η_m):

$$\eta_o = \frac{Shaft\ Power}{Water\ Power} = \frac{P}{\rho g\ Q\ H}$$

Relationship between efficiencies:

$$\eta_o = \eta_v \times \eta_h \times \eta_m$$

Problem from Pelton Turbine:

Problem-(1) *A Pelton wheel has a mean bucket speed of 10 metres per second with a jet of water flowing at the rate of 700 litres/s under a head of 30 metres. The buckets deflect the jet through an angle of 160°. Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume co-efficient of velocity as 0.98.*

Solution: Given :

Speed of bucket, $u = u_1 = u_2 = 10$ m/s

Discharge, $Q = 700$ litres/s $= 0.7$ m^3/s, Head of water, $H = 30$ m

Angle of deflection $= 160°$

∴ Angle, $\phi = 180° - 160° = 20°$

Co-efficient of velocity, $C_v = 0.98$.

The velocity of jet, $V_1 = C_v\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 30} = 23.77$ m/s

∴ $V_{r_1} = V_1 - u_1 = 23.77 - 10 = 13.77$ m/s

$V_{w_1} = V_1 = 23.77$ m/s

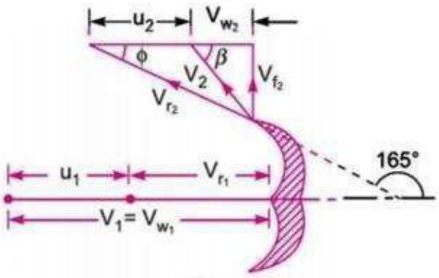


Fig. 18.6

From outlet velocity triangle,

$V_{r_2} = V_{r_1} = 13.77$ m/s

$V_{w_2} = V_{r_2}\cos\phi - u_2$

$= 13.77\cos 20° - 10.0 = 2.94$ m/s

Work done by the jet per second on the runner is given by equation (18.9) as

$= \rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u$

$= 1000 \times 0.7 \times [23.77 + 2.94] \times 10$ $\quad (\because\ aV_1 = Q = 0.7\ \text{m}^3/\text{s})$

$= 186970$ Nm/s

∴ Power given to turbine $= \dfrac{186970}{1000} =$ **186.97 kW. Ans.**

The hydraulic efficiency of the turbine is given by equation (18.12) as

$$\eta_h = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} = \frac{2[23.77 + 2.94] \times 10}{23.77 \times 23.77}$$

$$= \mathbf{0.9454} \text{ or } \mathbf{94.54\%.\ Ans.}$$

Problem (2) *A Pelton wheel is to be designed for the following specifications : Shaft power = 11,772 kW ; Head = 380 metres ; Speed = 750 r.p.m. ; Overall efficiency = 86% ; Jet diameter is not to exceed one-sixth of the wheel diameter. Determine :*

(i) The wheel diameter, *(ii) The number of jets required, and*

(iii) Diameter of the jet.

Take K_{v_1} = 0.985 and K_{u_1} = 0.45

Solution. Given :

Shaft power, S.P. = 11,772 kW

Head, $H = 380$ m

Speed, $N = 750$ r.p.m.

Overall efficiency, $\eta_0 = 86\%$ or 0.86

Ratio of jet dia. to wheel dia. $= \frac{d}{D} = \frac{1}{6}$

Co-efficient of velocity, $K_{v_1} = C_v = 0.985$

Speed ratio, $K_{u_1} = 0.45$

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 0.985\sqrt{2 \times 9.81 \times 380} = 85.05$ m/s

The velocity of wheel, $u = u_1 = u_2$

$= \text{Speed ratio} \times \sqrt{2gH} = 0.45 \times \sqrt{2 \times 9.81 \times 380} = 38.85$ m/s

But $u = \frac{\pi DN}{60} \quad \therefore \quad 38.85 = \frac{\pi DN}{60}$

or $D = \frac{60 \times 38.85}{\pi \times N} = \frac{60 \times 38.85}{\pi \times 750} = \mathbf{0.989\ m.\ Ans.}$

But $\frac{d}{D} = \frac{1}{6}$

$\therefore$ Dia. of jet, $d = \frac{1}{6} \times D = \frac{0.989}{6} = \mathbf{0.165\ m.\ Ans.}$

Discharge of one jet, q = Area of jet × Velocity of jet

$= \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4}(.165) \times 85.05 \text{ m}^3\text{/s} = 1.818 \text{ m}^3\text{/s}$ ...(*i*)

Now $\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \frac{11772}{\frac{\rho g \times Q \times H}{1000}}$

$0.86 = \frac{11772 \times 1000}{1000 \times 9.81 \times Q \times 380}$, where Q = Total discharge

$\therefore$ Total discharge, $Q = \frac{11772 \times 1000}{1000 \times 9.81 \times 380 \times 0.86} = 3.672 \text{ m}^3\text{/s}$

$\therefore$ Number of jets $= \frac{\text{Total discharge}}{\text{Discharge of one jet}} = \frac{Q}{q} = \frac{3.672}{1.818} = \mathbf{2\ jets.\ Ans.}$

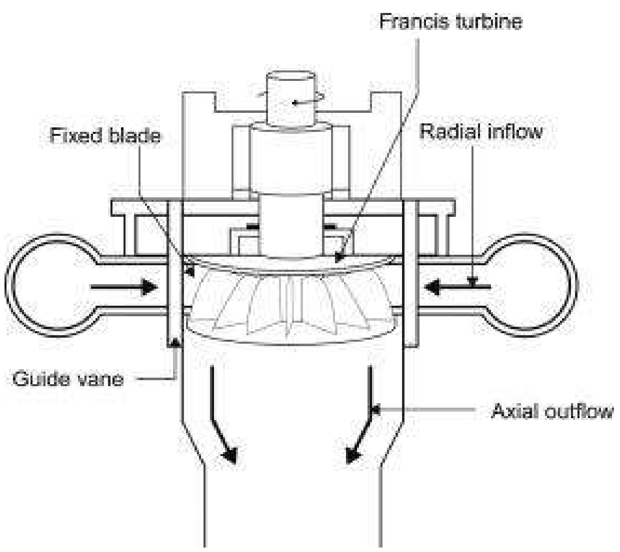
Reaction Turbine – Francis Turbine:

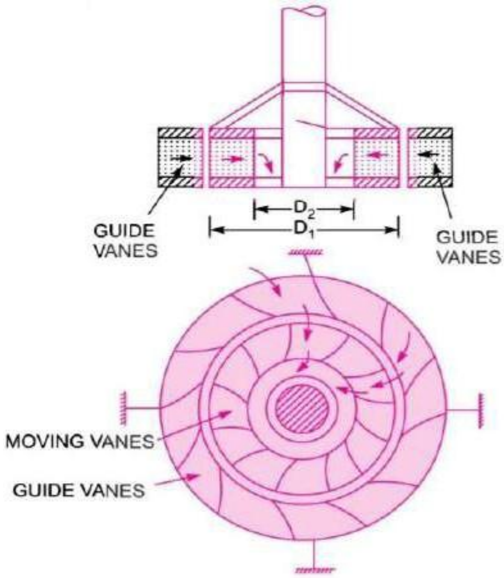
- Francis turbine is a medium head inward radial flow reaction turbine.
- Modern Francis turbine is an inward mixed flow reaction turbine. In this turbine, the water under pressure enters radially to the impeller blades while exits axially.
- When water flows radially from outward to inward, the turbine is called inward radial flow turbine
- When water flows radially from inward to outward, the turbine is called Outward radial flow turbine.
- An inward mixed flow reaction turbine, is a combination of impulse and reaction turbine where blades rotate using both reaction and impulse force of water flowing through them.

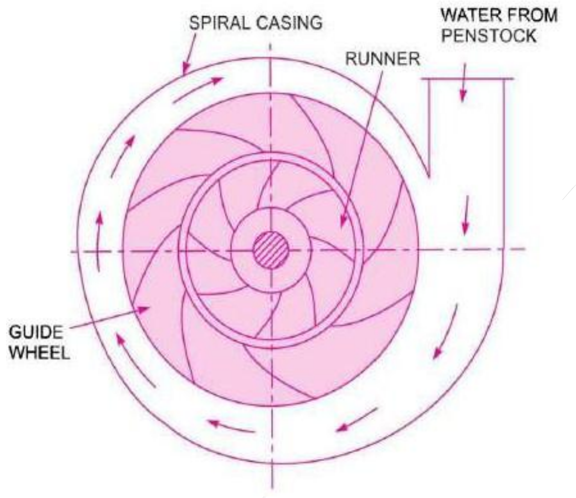
Construction of Francis Turbine:

Major Components component of Francis turbine are described below.

- **Spiral/Scroll Casing:**
  - Its cross-sectional area is maximum at inlet and minimum at exit.
  - It encloses the turbine runner completely and prevents the splashing of water.
  - It maintains constant velocity throughout the circumference.
- **Runner with fixed blades:**
  - It is a circular wheel with a series of radially curved vanes which are fixed on its periphery.
  - It provides rotational energy due to impulse and reaction effects on runner.
- **Penstock:**
  - It is the channels or pipelines which conveys water from source to the power station
- **Governing Mechanism:**
  - It controls the speed and power output of the turbine by changing the position of guide blades to vary the water flow rate at variation of loads.







(FRANCIS TURBINE)

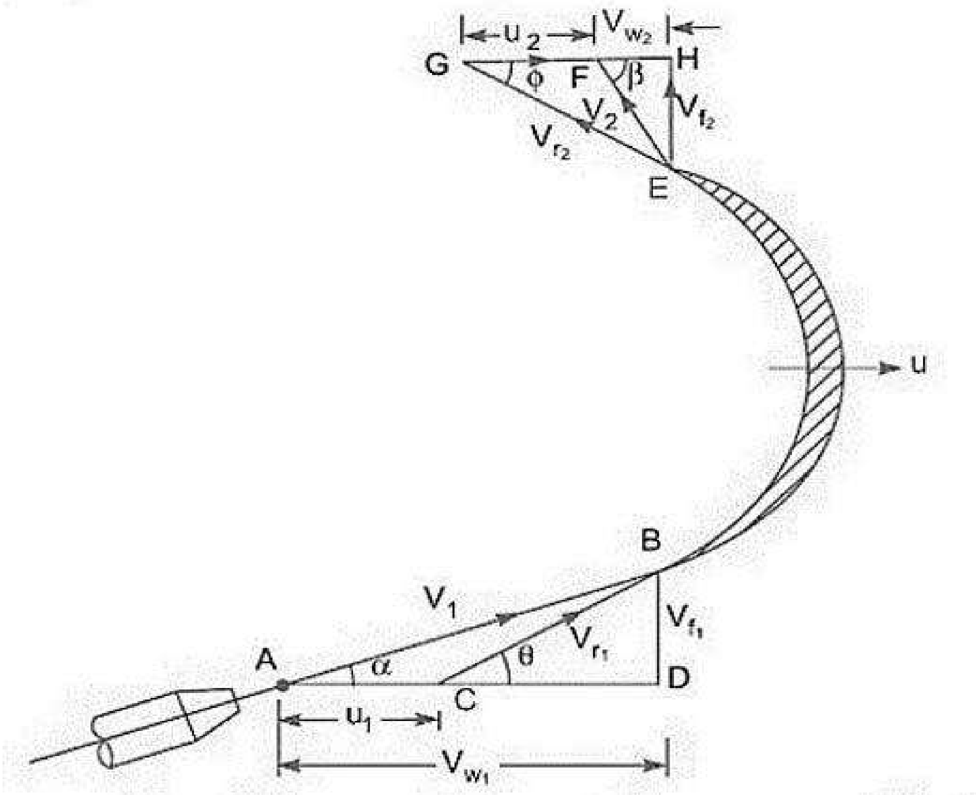
Working Principle:

- In modern Francis turbine; water enters into the turbine with both pressure and kinetic energy.
- When water flows through the stationary parts, a part of its pressure energy is converted into kinetic energy. When water flows over the moving parts, there is change in pressure, absolute velocity and direction.
- The pressure difference between the blade and runner is known as the reaction pressure. This pressure results the motion of the runner.

Velocity triangle of Francis turbine:

Consider the following terms for understanding the velocity triangle.

At inlet velocity triangle:	*At outlet velocity triangle:*
V_1 = absolute velocity of water	V_2 = absolute velocity of water
u_1 = peripheral velocity of runner (bucket speed)	u_2 = peripheral velocity of runner (bucket speed)
Vr_1 = relative velocity of water	Vr_2 = relative velocity of water
Vw_1 = velocity of whirl	Vw_2 = velocity of whirl
Vf_1 = velocity of flow	Vf_2 = velocity of flow
α = angle between the direction of the jet and the direction of motion of the vane (*guide blade angle*)	β = angle between the direction of the jet and the direction of motion of the vane (*guide blade angle*)
θ = angle made by the relative velocity Vr_1 with the direction of motion (*vane angle*)	φ = angle made by the relative velocity Vr2 with the direction of motion (*vane angle*)



(Velocity triangle of Francis turbine)

From inlet velocity triangle we obtain:	*From outlet velocity triangle we obtain:*
$u_1 = \pi D_1 N_1/60$	$u_2 = \pi D_2 N_2/60$
where D = diameter of wheel, N = speed in r.p.m	

Work done and power developed of a Pelton wheel:

Let, F = force exerted by the jet of water in the direction of motion

= mass x change in velocity in the direction of force

= m x ($V_{w1} + V_{w2}$) = $\rho a V_1$ x ($V_{w1} + V_{w2}$)

Where: ρ = density of water;

a = area of jet = $\frac{\pi}{4} \times d^2$

d = diameter of jet

Let, W = Net work done by the jet on runner per second

= $\rho a V_1$ x (V_{w1} x u_1 + V_{w2} x u_2)

Work done per second per unit weight of water striking = $\frac{\rho a V_1 \times (V_{w_1} u_1 + V_{w_2} u_2)}{\rho a V_1 \times g} = \frac{(V_{w_1} u_1 + V_{w_2} u_2)}{g}$

For radial discharge: $\beta = 90^0$ and $V_{w2} = 0$; Output is maximum

Therefore: Work done per second per unit weight of water striking = $\frac{V_{w_1} u_1}{g}$

Hydraulic efficiency:

$$\eta = \frac{Runner\ Power}{Water\ Power} = \frac{\rho a V_1 \times (Vw_1\, u_1 \pm Vw_2\, u_2)}{\rho g\, Q\, H} = \frac{Vw_1\, u_1 \pm Vw_2\, u_2}{g\ H}$$

For radial discharge: when $Vw_2 = 0$;

$$\eta_h = \frac{Vw_1\, u_1}{g\ H}$$

NOTE:

$$speed\ ratio = \frac{u_1}{\sqrt{2\, g\, H}}$$

$$flow\ ratio = \frac{V_{f_1}}{\sqrt{2\, g\, H}}$$

Discharge of the turbine = $Q = \pi \times D_1 \times B_1 \times Vf_1 = \pi \times D_2 \times B_2 \times Vf_2$

D_1 and D_2 are the diameter of runner at inlet and outlet respectively

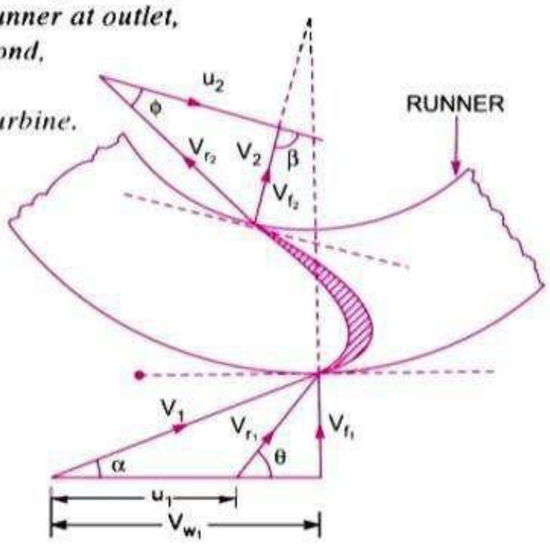
B_1 and B_2 are the width of runner at inlet and outlet respectively

Vf_1 and Vf_2 are the velocity of flow at the inlet and outlet respectively

Problems from Francis Turbine

Problem (3) *An inward flow reaction turbine has external and internal diameters as 0.9 m and 0.45 m respectively. The turbine is running at 200 r.p.m. and width of turbine at inlet is 200 mm. The velocity of flow through the runner is constant and is equal to 1.8 m/s. The guide blades make an angle of 10° to the tangent of the wheel and the discharge at the outlet of the turbine is radial. Draw the inlet and outlet velocity triangles and determine:*

(i) The absolute velocity of water at inlet of runner,
(ii) The velocity of whirl at inlet, (iii) The relative velocity at inlet,
(iv) The runner blade angles, (v) Width of the runner at outlet,
(vi) Mass of water flowing through the runner per second,
(vii) Head at the inlet of the turbine,
(viii) Power developed and hydraulic efficiency of the turbine.



Solution. Given :

External Dia., $D_1 = 0.9$ m
Internal Dia., $D_2 = 0.45$ m
Speed, $N = 200$ r.p.m.
Width at inlet, $B_1 = 200$ mm = 0.2 m
Velocity of flow, $V_{f_1} = V_{f_2} = 1.8$ m/s
Guide blade angle, $\alpha = 10°$
Discharge at outlet = Radial
$\therefore$ $\beta = 90°$ and $V_{w_2} = 0$

Tangential velocity of wheel at inlet and outlet are :

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times .9 \times 200}{60} = 9.424 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times .45 \times 200}{60} = 4.712 \text{ m/s.}$$

(i) Absolute velocity of water at inlet of the runner i.e., V_1

From inlet velocity triangle,

$$V_1 \sin \alpha = V_{f_1}$$

$$\therefore \quad V_1 = \frac{V_{f_1}}{\sin \alpha} = \frac{18}{\sin 10^\circ} = \textbf{10.365 m/s. Ans.}$$

(ii) Velocity of whirl at inlet, i.e., V_{w_1}

$$V_{w_1} = V_1 \cos \alpha = 10.365 \times \cos 10^\circ = \textbf{10.207 m/s. Ans.}$$

(iii) Relative velocity at inlet, i.e., V_{r_1}

$$V_{r_1} = \sqrt{V_{r_1}{}^3 + (V_{w_1} - u_1)^2} = \sqrt{1.8^2 + (10.207 - 9.424)^2}$$

$$= \sqrt{3.24 + .613} = \textbf{1.963 m/s. Ans.}$$

(iv) The runner blade angles means the angle θ and ϕ

Now $$\tan \theta = \frac{V_{f_1}}{\left(V_{w_1} - u_1\right)} = \frac{1.8}{(10.207 - 9.424)} = 2.298$$

$$\therefore \quad \theta = \tan^{-1} 2.298 = \mathbf{66.48^\circ} \text{ or } \mathbf{66^\circ\ 29'}. \textbf{ Ans.}$$

From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{u_2} = \frac{1.8}{4.712} = \tan 20.9^\circ$$

$$\therefore \quad \phi = \mathbf{20.9^\circ} \text{ or } \mathbf{20^\circ\ 54.4'}. \textbf{ Ans.}$$

(v) Width of runner at outlet, i.e., B_2

From equation (18.21), we have

$$\pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2} \text{ or } D_1 B_1 = D_2 B_2 \qquad \left(\because \quad \pi V_{f_1} = \pi V_{f_2} \text{ as } V_{f_1} = V_{f_2}\right)$$

$$\therefore \quad B_2 = \frac{D_1 B_1}{D_2} = \frac{0.90 \times 0.20}{0.45} = 0.40 \text{ m} = \textbf{400 mm. Ans.}$$

(vi) Mass of water flowing through the runner per second.

The discharge, $$Q = \pi D_1 B_1 V_{f_1} = \pi \times 0.9 \times 0.20 \times 1.8 = 1.0178 \text{ m}^3\text{/s.}$$

$$\therefore \quad \text{Mass} = \rho \times Q = 1000 \times 1.0178 \text{ kg/s} = \textbf{1017.8 kg/s. Ans.}$$

(vii) Head at the inlet of turbine, i.e., H.

Using equation (18.24), we have

$$H - \frac{V_2^2}{2g} = \frac{1}{g}(V_{w_1} u_1 \pm V_{w_2} u_2) = \frac{1}{g}(V_{w_1} u_1) \qquad (\because \text{ Here } V_{w_2} = 0)$$

$$H = \frac{1}{g} V_{w_1} u_1 + \frac{V_2^2}{2g} = \frac{1}{9.81} \times 10.207 \times 9.424 + \frac{1.8^2}{2 \times 9.81} \quad \left(\because V_2 = V_{f_2}\right)$$

$$= 9.805 + 0.165 = \textbf{9.97 m. Ans.}$$

(viii) Power developed, i.e., $$P = \frac{\text{Work done per second on runner}}{1000}$$

$$= \frac{\rho Q \left[V_{w_1} u_1\right]}{1000} \qquad \text{[Using equation (18.18)]}$$

$$= 1000 \times \frac{1.0178 \times 10.207 \times 9.424}{1000} = \textbf{97.9 kW. Ans.}$$

Hydraulic efficiency is given by equation (18.20*B*) as

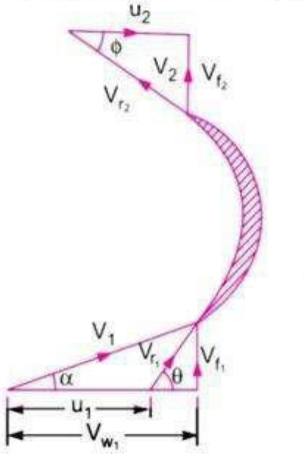
$$\eta_h = \frac{V_{w_1} u_1}{gH} = \frac{10.207 \times 9.424}{9.81 \times 9.97} = 0.9834 = \textbf{98.34\%. Ans.}$$

Problem (4) *A reaction turbine works at 450 r.p.m. under a head of 120 metres. Its diameter at inlet is 120 cm and the flow area is 0.4 m². The angles made by absolute and relative velocities at inlet are 20° and 60° respectively with the tangential velocity. Determine :*

(a) The volume flow rate, (b) The power developed, and

(c) Hydraulic efficiency.

Assume whirl at outlet to be zero.



Solution. Given :

Speed of turbine, $N = 450$ r.p.m.

Head, $H = 120$ m

Diameter at inlet, $D_1 = 120 \text{ cm} = 1.2 \text{ m}$

Flow area, $\pi D_1 \times B_1 = 0.4 \text{ m}^2$

Angle made by absolute velocity at inlet, $\alpha = 20°$

Angle made by the relative velocity at inlet, $\theta = 60°$

Whirl at outlet, $V_{w_2} = 0$

Tangential velocity of the turbine at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.2 \times 450}{60} = 28.27 \text{ m/s}$$

From inlet velocity triangle,

$$\tan \alpha = \frac{V_{f_1}}{V_{w_1}} \text{ or } \tan 20° = \frac{V_{f_1}}{V_{w_1}} \text{ or } \frac{V_{f_1}}{V_{w_1}} = \tan 20° = 0.364$$

$\therefore$ $$V_{f_1} = 0.364\ V_{w_1}$$

Also $$\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{0.364\ V_{w_1}}{V_{w_1} - 28.27} \qquad (\because \; V_{f_1} = 0.364\ V_{w_1})$$

or $$\frac{0.364\ V_{w_1}}{V_{w_1} - 28.27} = \tan \theta = \tan 60° = 1.732$$

$\therefore$ $$0.364 V_{w_1} = 1.732(V_{w_1} - 28.27) = 1.732 V_{w_1} - 48.96$$

or $$(1.732 - 0.364)\ V_{w_1} = 48.96$$

$\therefore$ $$V_{w_1} = \frac{48.96}{(1.732 - 0.364)} = 35.789 = 35.79 \text{ m/s.}$$

From equation (*i*), $$V_{f_1} = 0.364 \times V_{w_1} = 0.364 \times 35.79 = 13.027 \text{ m/s.}$$

(*a*) Volume flow rate is given by equation (18.21) as $Q = \pi D_1 B_1 \times V_{f_1}$

But $\pi D_1 \times B_1 = 0.4 \text{ m}^2$ (given)

$$Q = 0.4 \times 13.027 = \mathbf{5.211\ m^3/s.\ Ans.}$$

(*b*) Work done per sec on the turbine is given by equation (18.18),

$$= \rho Q\ [V_{w_1} u_1] \qquad \left(\because \; V_{w_2} = 2\right)$$

$$= 1000 \times 5.211\ [35.79 \times 28.27] = 5272402 \text{ Nm/s}$$

$\therefore$ Power developed in kW $= \dfrac{\text{Work done per second}}{1000} = \dfrac{5272402}{1000} = \mathbf{5272.402\ kW.\ Ans.}$

(*c*) The hydraulic efficiency is given by equation (18.20*B*) as

$$\eta_h = \frac{V_{w_1} u_1}{gH} = \frac{35.79 \times 28.27}{9.81 \times 120} = 0.8595 = \mathbf{85.95\%.\ Ans.}$$

Problem (5) *As inward flow reaction turbine has external and internal diameters as 1.0 m and 0.6 m respectively. The hydraulic efficiency of the turbine is 90% when the head on the turbine is 36 m. The velocity of flow at outlet is 2.5 m/s and discharge at outlet is radial. If the vane angle at outlet is 15° and width of the wheel is 100 mm at inlet and outlet, determine : (i) the guide blade angle, (ii) speed of the turbine, (iii) vane angle of the runner at inlet, (iv) volume flow rate of turbine and (v) power developed.*

Solution. Given :

External diameter, $D_1 = 1.0$ m

Internal diameter, $D_2 = 0.6$ m

Hydraulic efficiency, $\eta_h = 90\% = 0.90$

Head, $H = 36$ m

Velocity of flow at outlet, $V_{f_2} = 2.5$ m/s

Discharge is radial, $V_{w_2} = 0$

Vane angle at outlet, $\phi = 15°$

Width of wheel, $B_1 = B_2 = 100$ mm $= 0.1$ m

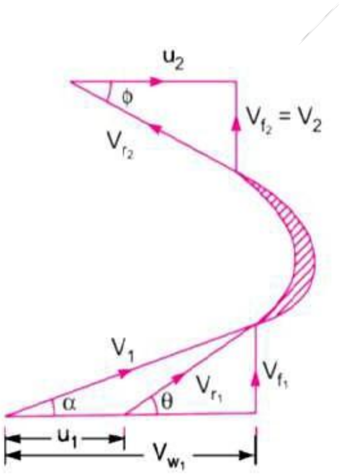


Fig. 18.14

Using equation (18.20 *B*) for hydraulic efficiency as

$$\eta_h = \frac{V_{w_1} u_1}{gH} \text{ or } 0.90 = \frac{V_{w_1} . u_1}{9.81 \times 36}$$

$\therefore$ $$V_{w_1} u_1 = 0.90 \times 9.81 \times 36 = 317.85 \quad ...(i)$$

From outlet velocity triangle, $\tan \phi = \frac{V_{f_2}}{u_2} = \frac{2.5}{u_2}$

$\therefore$ $$u_2 = \frac{2.5}{\tan \phi} = \frac{2.5}{\tan 15°} = 9.33$$

But $$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times N}{60}$$

$\therefore$ $$9.33 = \frac{\pi \times 0.6 \times N}{60} \text{ or } N = \frac{60 \times 9.33}{\pi \times 0.6} = \mathbf{296.98.\ Ans.}$$

$\therefore$ $$u_1 = \frac{\pi \times D_1 \times N}{60} = \frac{\pi \times 1.0 \times 296.98}{60} = 15.55 \text{ m/s.}$$

Substituting this value of 'u_1' in equation (*i*),

$$V_{w_1} \times 15.55 = 317.85$$

$\therefore$ $$V_{w_1} = \frac{317.85}{15.55} = 20.44 \text{ m/s}$$

Using equation (18.21), $\pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2}$ or $D_1 V_{f_1} = D_2 V_{f_2}$ $(\because B_1 = B_2)$

$\therefore$ $$V_{f_1} = \frac{D_2 \times V_{f_2}}{D_1} = \frac{0.6 \times 2.5}{1.0} = 1.5 \text{ m/s.}$$

(*i*) *Guide blade angle* (α).

From inlet velocity triangle, $\tan \alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{1.5}{20.44} = 0.07338$

$\therefore \quad \alpha = \tan^{-1} 0.07338 = \mathbf{4.19°}$ or $\mathbf{4° \, 11.8'}$. **Ans.**

(*ii*) *Speed of the turbine,* $N = \mathbf{296.98}$ **r.p.m. Ans.**

(*iii*) *Same angle of runner at inlet* (θ)

$$\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{1.5}{(20.44 - 15.55)} = 0.3067$$

$\therefore \quad \theta = \tan^{-1} .3067 = 17.05°$ or $\mathbf{17° \, 3'}$. **Ans.**

(*iv*) *Volume flow rate of turbine* is given by equation (18.21) as

$$= \pi D_1 B_1 V_{f_1} = \pi \times 1.0 \times 0.1 \times 1.5 = \mathbf{0.4712 \ m^3/s.} \ \textbf{Ans.}$$

(*v*) *Power developed* (in kW)

$$= \frac{\text{Work done per second}}{1000} = \frac{\rho Q \left[V_{w_1} u_1\right]}{1000}$$

[Using equation (18.18) and $V_{w_2} = 0$]

$$= 1000 \times \frac{0.4712 \times 20.44 \times 15.55}{1000} = \mathbf{149.76 \ kW.} \ \textbf{Ans.}$$

Axial flow reaction Turbine:

- In an axial flow reaction turbine water flows parallel to the axis of rotation of the shaft of the turbine.
- It has a vertical shaft with larger lower end known as hub/boss.
- Vanes are fixed on the hub and so the hub works like a runner.
- It requires large quantity of water at low head

Classification of Axial flow reaction Turbine:

Axial flow reactions are classified as:

- Propellor turbine
  - Propeller turbine is the axial flow reaction turbine which has not adjustable fixed vanes.
- Kaplan turbine
  - Kaplan turbine is the axial flow reaction turbine which has adjustable vanes.

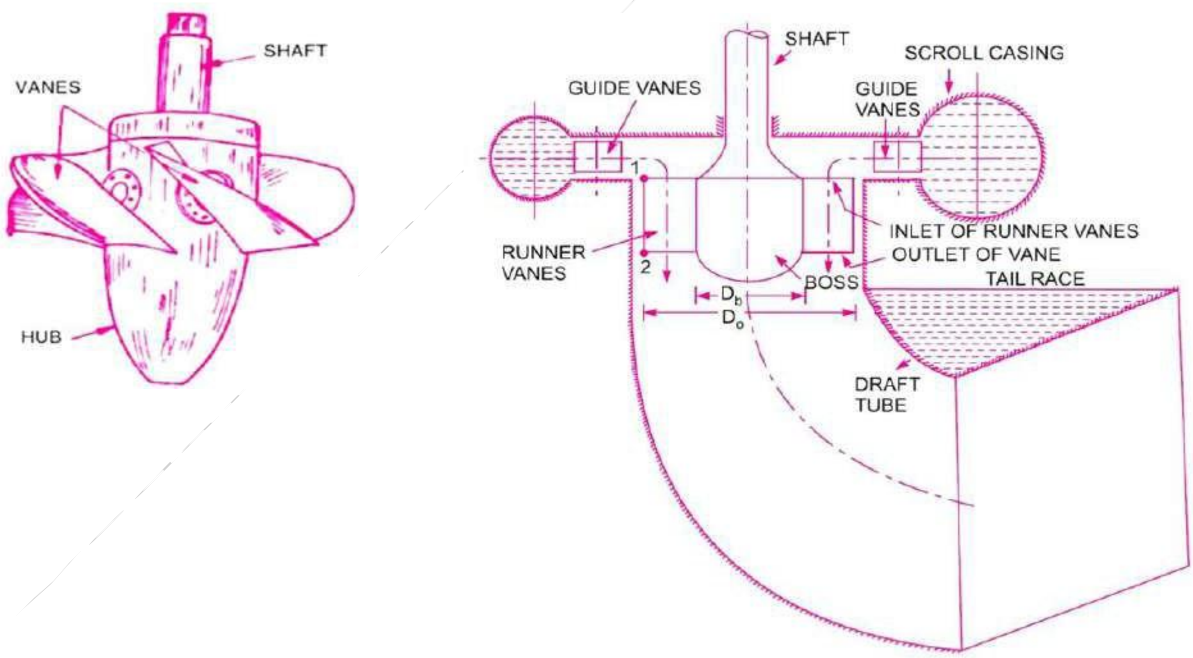
Kaplan Turbine:

- It is an axial flow reaction turbine in which water flows parallel to the axis of rotation of the shaft of the turbine. The water enters the runner of turbine in an axial direction and leaves the runner axially.
- It has a vertical shaft with larger lower end known as hub/boss.
- Vanes are fixed on the hub and so the hub works like a runner.
- It requires large quantity of water at low head

Construction of Kaplan Turbine:

It consists of the following major parts.

- Scroll Casing:
  - It encloses the turbine runner completely and prevents the splashing of water.
  - Cross-section of scroll casing decreases uniformly to maintain the pressure of water such that the flow pressure is not lost.
  - From the scroll casing the guide vanes direct the water to the runner.
- Guide vanes mechanism:
  - The guide vanes are adjustable and can be adjusted to meet the required flow rate.
  - Guide vanes also control the swirl of the water flow.
- Hub with vanes:
  - The vanes are fixed on the hub and hence hub acts as a runner for the axial flow reaction turbine.
- Draft tube:
  - The draft tube is a connecting pipe whose inlet is fitted at the outlet of the turbine.
  - The diameter of the draft tube is small near its inlet and large near its outlet. The outlet of the draft tube is always submerged in water.
  - It **c**onverts the kinetic energy of the water to static pressure at the outlet of the turbine**.** So pressure of the exit fluid increases. This helps to avoid the dissipation of the kinetic energy of the exit water. It improves the capacity of the turbine.



(KAPLAN TURBINE)

Key points for Kaplan Turbine:

1. Discharge through the runner: $Q = \frac{\pi}{4} \times (D_o^2 - D_b^2) \times V_{f1}$

   Where: D_o = diameter of the runner

   D_b = diameter of the hub/boss

   V_{f1} = velocity of flow at inlet
2. Area of flow at inlet = Area of flow at outlet = $A = \frac{\pi}{4} \times (D_o^2 - D_b^2)$
3. Peripheral velocity at inlet and outlet are equal: $u_1 = u_2 = \frac{\pi D_o N}{60}$
4. Velocity of flow at inlet (V_{f1}) = Velocity of flow at outlet (V_{f2})
5. Speed ratio = $\frac{u^1}{\sqrt{2gH}}$
6. Flow ratio = $\frac{V_{f1}}{\sqrt{2gH}}$
7. Water power = $\frac{\rho g Q H}{1000}$
8. Runner Power = $\frac{1}{g} \times \frac{V_{w1}u_1 + V_{w1}u_2}{1000}$
9. Hydraulic efficiency = $\frac{V_{w1}u_1}{gH}$ (for radial discharge)
10. Overall efficiency = $\frac{SP}{WP}$
11. Specific speed pf turbine = $N_s = \frac{N\sqrt{P}}{H^{5/4}}$

Problems from Kaplan Turbine:

Problem (6) *A Kaplan turbine working under a head of 20 m develops 11772 kW shaft power. The outer diameter of the runner is 3.5 m and hub diameter is 1.75 m. The guide blade angle at the extreme edge of the runner is 35°. The hydraulic and overall efficiencies of the turbines are 88% and 84% respectively. If the velocity of whirl is zero at outlet, determine :*

(i) Runner vane angles at inlet and outlet at the extreme edge of the runner, and

(ii) Speed of the turbine.

Solution. Given :

Head,	$H = 20$ m
Shaft power,	S.P. = 11772 kW
Outer dia. of runner,	$D_o = 3.5$ m
Hub diameter,	$D_b = 1.75$ m
Guide blade angle,	$\alpha = 35°$
Hydraulic efficiency,	$\eta_h = 88\%$
Overall efficiency,	$\eta_o = 84\%$
Velocity of whirl at outlet	= 0.

Using the relation, $\eta_o = \frac{\text{S.P.}}{\text{W.P.}}$

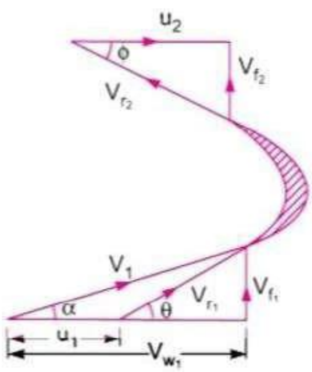


Fig. 18.27

$$\text{W.P.} = \frac{\text{W.P.}}{1000} = \frac{\rho \times g \times Q \times H}{1000}, \text{ we get}$$

$$0.84 = \frac{11772}{\frac{\rho \times g \times Q \times H}{1000}}$$

$$= \frac{11772 \times 1000}{1000 \times 9.81 \times Q \times 20} \qquad (\because \rho = 1000)$$

$$\therefore \quad Q = \frac{11772 \times 1000}{0.84 \times 1000 \times 9.81 \times 20} = 71.428 \text{ m}^3\text{/s.}$$

Using equation (18.25), $Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_{f_1}$

or $71.428 = \frac{\pi}{4}(3.5^2 - 1.75^2) \times V_{f_1} = \frac{\pi}{4}(12.25 - 3.0625)\ V_{f_1}$

$$= 7.216\ V_{f_1}$$

$$\therefore \quad V_{f_1} = \frac{71.428}{7.216} = 9.9 \text{ m/s.}$$

From inlet velocity triangle, $\tan \alpha = \frac{V_{f_2}}{V_{w_1}}$

$$\therefore \quad V_{w_1} = \frac{V_{f_1}}{\tan \alpha} = \frac{9.9}{\tan 35^\circ} = \frac{9.9}{.7} = 14.14 \text{ m/s}$$

Using the relation for hydraulic efficiency,

$$\eta_h = \frac{V_{w_1} u_1}{gH} \qquad \left(\because \ V_{w_2} = 0\right)$$

$$0.88 = \frac{14.14 \times u_1}{9.81 \times 20}$$

$$\therefore \quad u_1 = \frac{0.88 \times 9.81 \times 20}{14.14} = 12.21 \text{ m/s.}$$

(*i*) Runner vane angles at inlet and outlet at the extreme edge of the runner are given as :

$$\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{9.9}{(14.14 - 12.21)} = 5.13$$

$$\therefore \quad \theta = \tan^{-1} 5.13 = \mathbf{78.97^\circ} \text{ or } \mathbf{78^\circ\ 58'}. \ \textbf{Ans.}$$

For Kaplan turbine, $u_1 = u_2 = 12.21$ m/s and $V_{f_1} = V_{f_2} = 9.9$ m/s

$\therefore$ From outlet velocity triangle, $\tan \phi = \frac{V_{f_2}}{u_2} = \frac{9.9}{12.21} = 0.811$

$$\therefore \quad \phi = \tan^{-1} .811 = \mathbf{39.035^\circ} \text{ or } \mathbf{39^\circ\ 2'}. \ \textbf{Ans.}$$

(*ii*) Speed of turbine is given by $u_1 = u_2 = \frac{\pi D_o N}{60}$

$$12.21 = \frac{\pi \times 3.5 \times N}{60}$$

$$\therefore \quad N = \frac{60 \times 12.21}{\pi \times 3.50} = \textbf{66.63 r.p.m. Ans.}$$

Problem (7) *A Kaplan turbine develops 24647.6 kW power at an average head of 39 metres. Assuming a speed ratio of 2, flow ratio of 0.6, diameter of the boss equal to 0.35 times the diameter of the runner and an overall efficiency of 90%, calculate the diameter, speed and specific speed of the turbine.*

Solution. Given :

Shaft power, $S.P. = 24647.6$ kW

Head, $H = 39$ m

Speed ratio, $u_1\sqrt{2gH} = 2.0$

$\therefore \quad u_1 = 2.0 \times \sqrt{2gH} = 2.0 \times \sqrt{2 \times 9.81 \times 39} = 55.32$ m/s

Flow ratio, $\dfrac{V_{f_1}}{\sqrt{2gH}} = 0.6$

$\therefore \quad V_{f_1} = 0.6 \times \sqrt{2gH} = 0.6 \times \sqrt{2 \times 9.81 \times 39} = 16.59$ m/s

Diameter of boss $= 0.35 \times$ Diameter of runner

$\therefore \quad D_b = 0.35 \times D_o$

Overall efficiency, $\eta_o = 90\% = 0.90$

Using the relation, $\eta_o = \dfrac{\text{S.P.}}{\text{W.P.}}$, where $\text{W.P.} = \dfrac{\rho \times g \times Q \times H}{1000}$

$$\therefore \quad 0.90 = \frac{24647.6}{\dfrac{\rho \times g \times Q \times H}{1000}} = \frac{24647.6 \times 1000}{1000 \times 9.81 \times Q \times 39}$$

$$\therefore \quad Q = \frac{24647.6 \times 1000}{0.9 \times 1000 \times 9.81 \times 39} = 71.58 \text{ m}^3/\text{s}.$$

But from equation (18.25), we have

$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_{f_1}$$

$$\therefore \quad 71.58 = \frac{\pi}{4}\left[D_o^2 - (.35\,D_o)^2\right] \times 16.59 \qquad (\because \; D_b = 0.35\,D_o,\; V_{f_1} = 16.59)$$

$$= \frac{\pi}{4}\left[D_o^2 - .1225\,D_o^2\right] \times 16.59$$

$$= \frac{\pi}{4} \times .8775\,D_o^2 \times 16.59 = 11.433\,D_o^2$$

(*i*) $\therefore \quad D_o = \sqrt{\dfrac{71.58}{11.433}} = $ **2.5 m. Ans.**

$\therefore \quad D_b = 0.35 \times D_o = 0.35 \times 2.5 = $ **0.875 m. Ans.**

(*ii*) Speed of the turbine is given by $u_1 = \dfrac{\pi D_o N}{60}$

$$\therefore \quad 55.32 = \frac{\pi \times 2.5 \times N}{60}$$

$\therefore \quad N = \dfrac{60 \times 55.32}{\pi \times 2.5} = $ **422.61 r.p.m. Ans.**

(*iii*) Specific speed * is given by $N_s = \dfrac{N\sqrt{P}}{H^{5/4}}$, where P = Shaft power in kW

$\therefore \quad N_s = \dfrac{422.61 \times \sqrt{24647.6}}{(39)^{5/4}} = \dfrac{422.61 \times 156.99}{97.461} = $ **680.76 r.p.m. Ans.**

Problem (8) *The hub diameter of a Kaplan turbine, working under a head of 12 m, is 0.35 times the diameter of the runner. The turbine is running at 100 r.p.m. If the vane angle of the extreme edge of the runner at outlet is 15° and flow ratio is 0.6, find :*

(i) Diameter of the runner, (ii) Diameter of the boss, and

(iii) Discharge through the runner.

The velocity of whirl at outlet is given as zero.

Solution. Given :

Head, $H = 12$ m

Hub diameter, $D_b = 0.35 \times D_o$, where D_o = Dia. of runner

Speed, $N = 100$ r.p.m.

Vane angle at outlet, $\phi = 15°$

Flow ratio $= \dfrac{V_{f_1}}{\sqrt{2gH}} = 0.6$

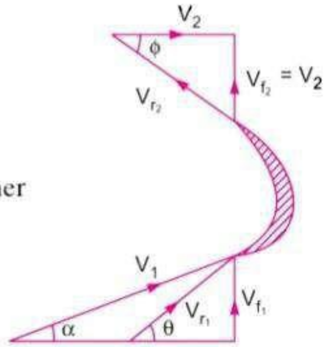


Fig. 18.28

$$\therefore \quad V_{f_1} = 0.6 \times \sqrt{2gH} = 0.6 \times \sqrt{2 \times 9.81 \times 12} = 9.2 \text{ m/s.}$$

From the outlet velocity triangle, $V_{w_2} = 0$

$$\tan\phi = \frac{V_{f_2}}{u_2} = \frac{V_{f_1}}{u_2}$$

$$\therefore \quad \tan 15° = \frac{9.2}{u_2}$$

$$\therefore \quad u_2 = \frac{9.2}{\tan 15°} = 34.33 \text{ m/s.}$$

But for Kaplan turbine, $u_1 = u_2 = 34.33$

Now, using the relation, $u_1 = \dfrac{\pi D_o \times N}{60}$ or $34.33 = \dfrac{\pi \times D_o \times 100}{60}$

$$D_o = \frac{60 \times 34.33}{\pi \times 100} = \mathbf{6.55 \text{ m. Ans.}}$$

$$\therefore \quad D_b = 0.35 \times D_o = 0.35 \times 6.35 = \mathbf{2.3 \text{ m. Ans.}}$$

Discharge through turbine is given by equation (18.25) as

$$Q = \frac{\pi}{4}\left[D_o^2 - D_b^2\right] \times V_{f_1} = \frac{\pi}{4}\left[6.55^2 - 2.3^2\right] \times 9.2$$

$$= \frac{\pi}{4}(42.9026 - 5.29) \times 9.2 = \mathbf{271.77 \text{ m}^3\text{/s. Ans.}}$$

Difference between Impulse and Reaction turbine:

Impulse Turbine	Reaction Turbine
The water flows through the nozzles and impinges on the moving blades	The water flows first through guide mechanism and then through the moving blades
The water impinges on the buckets with kinetic energy	The water glides over the moving vanes with pressure and kinetic energy
The water may or may not be admitted over the whole circumference.	The water must be admitted over the whole circumference
The water pressure remains constant during its flow through the moving blades.	The water pressure is reduced during its flow through the moving blades.
The relative velocity of water while gliding over the blades remains constant.	The relative velocity of water while gliding over the moving blades increase
The blades are symmetrical	The blades are not symmetrical
The number of stages required is less for the same power developed.	The number of stages required is more for the same power developed

CENTRIGUGAL PUMP

Introduction:

- It is a hydraulic machine in which force is transmitted by means of motion of fluid under pressure.
- In this machine, mechanical energy is converted into hydraulic energy in the form of pressure energy by the action of centrifugal force on the fluid.
- Its main purpose is to transfer fluids through an increase in pressure.
- It acts as a reverse of an inward flow reaction turbine.
- It is used in the field of agriculture, municipality, industries, power plants, petrochemicals, mining etc.

Construction:

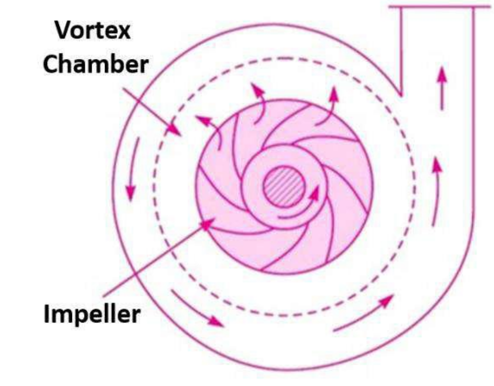
Major components of Centrifugal pump are:

- **Casing:**
  It is an air tight passage which surrounds the impeller. It converts kinetic energy of water into pressure energy with its special design. It is classified as:
  - Volute casing
    - *Volute casing* is the spiral casing in which the area of flow increases from inlet to outlet. This gradual increase in area helps to reduce the velocity of flow and increase the pressure at outlet. Due to formation of eddies, there is a limitation of energy loss.
  - Vortex casing
    - In *Vortex casing* a circular chamber is provided in between the impeller and casing. This decreases the energy loss formation of eddies. It helps to increase the efficiency of the pump.
  - Casing with guide blades
    - In *Casing with guide blades* a series of guide blades mounted on a ring surrounds the impeller. This helps to control the velocity and pressure of water by adjusting the guide blades.
- **Impeller:**

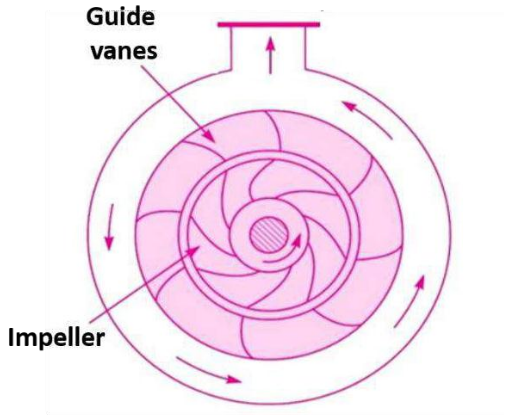
  It is a wheel or rotor which is provided with a series of backward curved blades or vanes. It is mounted on the shaft powered by motor.
- **Suction pipe with foot valve and strainer:**

  It's one end connects the inlet of the impeller and the other end is dipped into the sump of water. The foot valve fitted to the bottom of suction pipe is a one way valve that opens in the upward direction. The strainer fitted to the bottom of suction pipe is used to filter the unwanted particle present in the water to prevent the centrifugal pump from blockage.
- **Delivery pipe and delivery valve:**

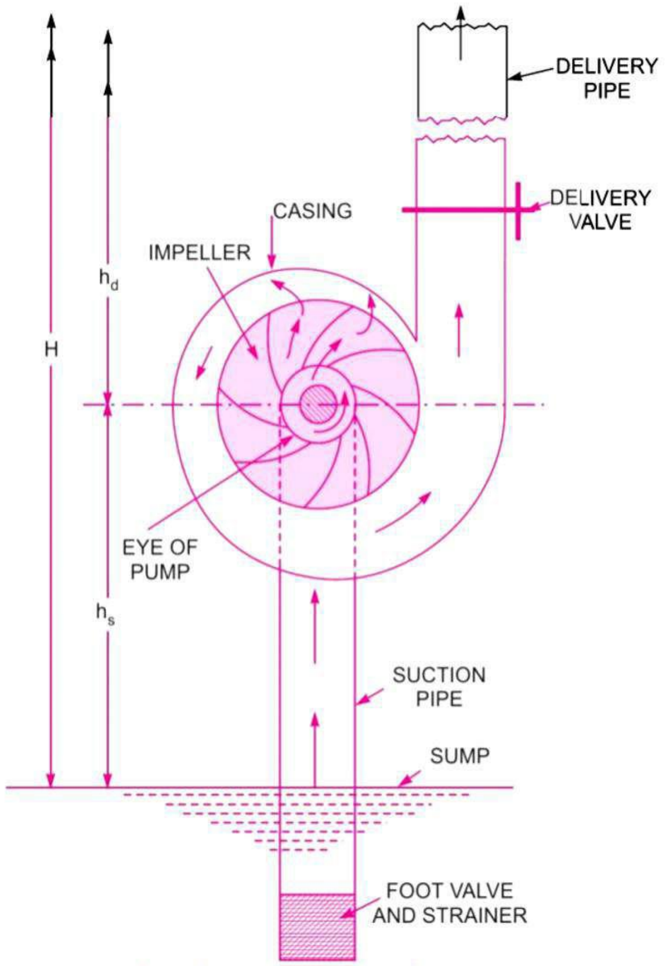
  It's one end connects the outlet of the pump and other end connects the point where water is delivered. A delivery valve is fitted with the outlet controls the flow from the pump into delivery pipe.



Vortex Casing

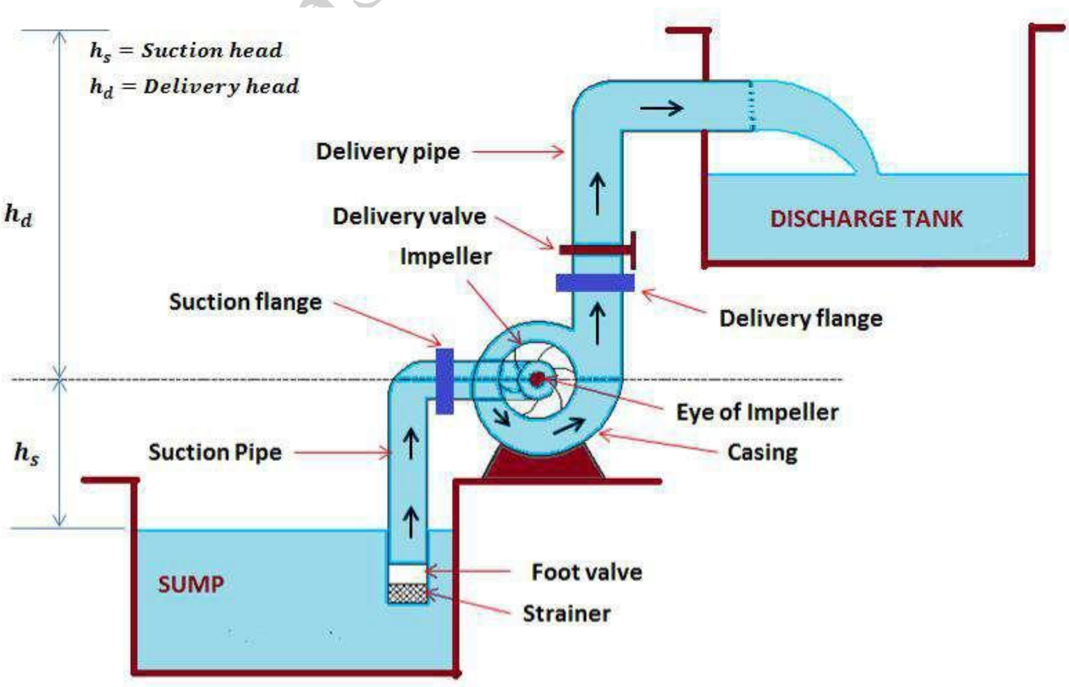


Casing with guide vanes



Centrifugal pump with Volute Casing

Working of Centrifugal Pump:

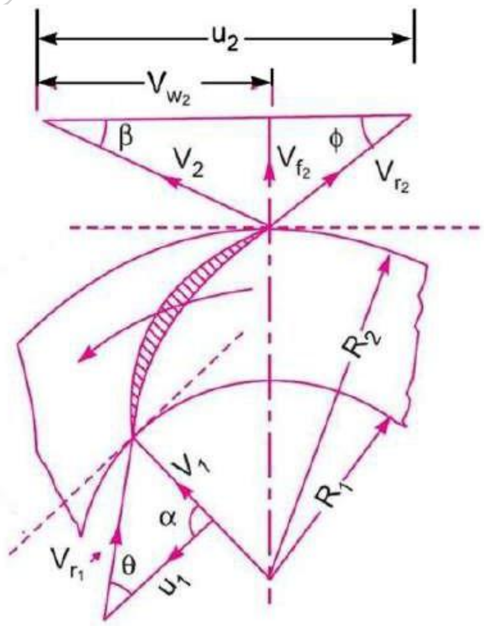


- When the electric motor starts, the shaft of the pump coupled with the motor shaft rotates. It gives rotational motion to the impeller mounted on the shaft.
- The rotating impeller drives the water inside it and produce centrifugal force. This creates velocity difference between the inlet and outlet.
- It causes the rising of water from sump through suction pipe to eye of the impeller.
- When water gets pressurized, the delivery valve opens to discharge water to desired height.
- **Priming** is the operation in which water is feed into the casing and suction pipeline keeping the delivery valve closed, so that all the air from the pump is driven out and no air is left.

Velocity triangle of Centrifugal pump:

Consider the following terms for understanding the velocity triangle.

At inlet velocity triangle:	*At outlet velocity triangle:*
V_1 = absolute velocity of water	V_2 = absolute velocity of water
u_1 = peripheral velocity of runner (bucket speed)	u_2 = peripheral velocity of runner (bucket speed)
Vr_1 = relative velocity of water	Vr_2 = relative velocity of water
Vw_1 = velocity of whirl	Vw_2 = velocity of whirl
Vf_1 = velocity of flow	Vf_2 = velocity of flow
α = angle between the direction of the jet and the direction of motion of the vane (*guide blade angle*)	β = angle between the direction of the jet and the direction of motion of the vane (*guide blade angle*)
θ = angle made by the relative velocity Vr_1 with the direction of motion (*vane angle*)	φ = angle made by the relative velocity Vr2 with the direction of motion (*vane angle*)



When water enters the impeller radially. at inlet

$\alpha = 90^0$, $Vw_1 = 0$, $Vf_1 = V_1$

$$u_1 = \frac{\pi D_1 N_1}{60} \quad and \quad u_2 = \frac{\pi D_2 N_2}{60}$$

$$Volume of water per\ (Q) = \pi D_1 N_1 V_{f_1} = \pi D_2 N_2 V_{f_2}$$

Let, W = Net work done by the jet on runner per second = $\rho a V_1$ x (Vw_2 x u_2)

Work done per second per unit weight of water striking = (Vw_2 x u_2)/g

Heads of Centrifugal pump:

Suction head (h_s)

It is the vertical distance between the centre line of pump and the water surface at sump level.

Delivery head (h_d)

It is the vertical distance between the centre line of pump and the water surface at the discharge tank.

Static head (H)

It is the sum of suction and delivery head.

Manometric head (H_m)

It is the working head of the centrifugal pump.

It is given by:

H_m **= (**Head imparted by impeller to the water – loss of head in the pump)

$$H_m = \frac{(Vw_2 \times u_2)}{g} - loss\ of\ head$$

If loss of head is neglected

$$H_m = \frac{(Vw_2 \times u_2)}{g}$$

Efficiencies of Centrifugal pump:

Manometric efficiency (η_{man})

It is the ratio between manometric head and the head imparted by the impeller to the water.

$$\eta_{man} = \frac{H_m}{\left(\frac{Vw_2 \times u_2}{g}\right)} = \frac{g H_m}{Vw_2 \times u_2}$$

Mechanical efficiency (η_m)

It is the ratio between the power at the impeller and the power at the shaft.

$$\eta_m = \frac{\rho \times Q \times Vw_2 \times u_2}{S.P}$$

Overall efficiency (η_o)

It is the ratio between the power output of the pump and the power input of the pump.

$$\eta_o = \frac{S.P}{\left(\frac{g H \; \square_m}{Vw_2 \times u_2} \right)}$$

Relation between η_{man}, η_m & η_o

$$\eta_o = \eta_{man}$$

Problem from Pelton Turbine:

Problem (1) *The internal and external diameters of the impeller of a centrifugal pump are 200 mm and 400 mm respectively. The pump is running at 1200 r.p.m. The vane angles of the impeller at inlet and outlet are 20° and 30° respectively. The water enters the impeller radially and velocity of flow is constant. Determine the work done by the impeller per unit weight of water.*

Solution. Given :

Internal diameter of impeller, $D_1 = 200$ mm $= 0.20$ m

External diameter of impeller, $D_2 = 400$ mm $= 0.40$ m

Speed, $N = 1200$ r.p.m.

Vane angle at inlet, $\theta = 20°$

Vane angle at outlet, $\phi = 30°$

Water enters radially* means, $\alpha = 90°$ and $V_{w_1} = 0$

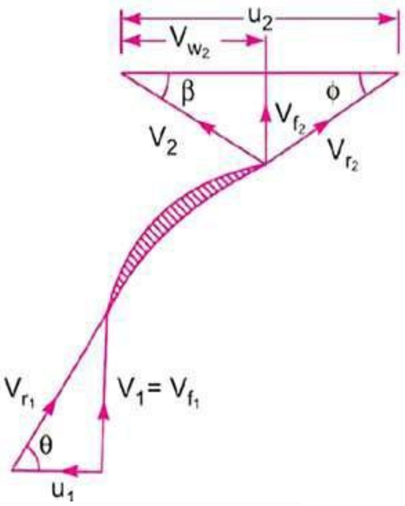
Velocity of flow, $V_{f_1} = V_{f_2}$

Tangential velocity of impeller at inlet and outlet are,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.20 \times 1200}{60} = 12.56 \text{ m/s}$$

and

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 1200}{60} = 25.13 \text{ m/s.}$$



From inlet velocity triangle, $\tan\theta = \frac{V_{f_1}}{u_1} = \frac{V_{f_1}}{12.56}$

$\therefore \quad V_{f_1} = 12.56 \tan\theta = 12.56 \times \tan 20° = 4.57$ m/s

$\therefore \quad V_{f_2} = V_{f_1} = 4.57$ m/s.

From outlet velocity triangle, $\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{4.57}{25.13 - V_{w_2}}$

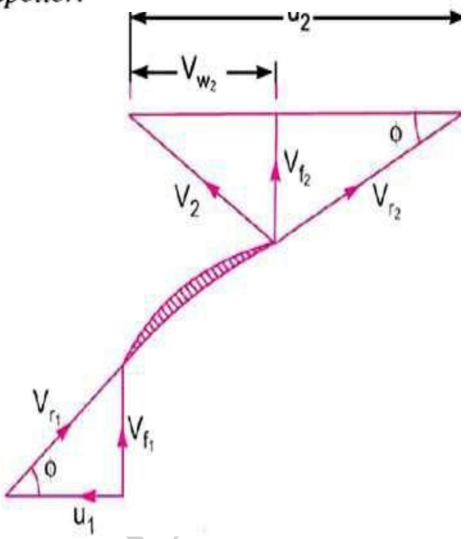
or $\quad 25.13 - V_{w_2} = \frac{4.57}{\tan\phi} = \frac{4.57}{\tan 30°} = 7.915$

$\therefore \quad V_{w_2} = 25.13 - 7.915 = 17.215$ m/s.

The work done by impeller per kg of water per second is given by equation (19.1) as

$$= \frac{1}{g} V_{w_2} u_2 = \frac{17.215 \times 25.13}{9.81} = \textbf{44.1 Nm/N. Ans.}$$

Problem (2) *A centrifugal pump is to discharge 0.118 m³/s at a speed of 1450 r.p.m. against a head of 25 m. The impeller diameter is 250 mm, its width at outlet is 50 mm and manometric efficiency is 75%. Determine the vane angle at the outer periphery of the impeller.*



Solution. Given :

Discharge, $Q = 0.118\ m^3/s$

Speed, $N = 1450$ r.p.m.

Head, $H_m = 25$ m

Diameter at outlet, $D_2 = 250$ mm $= 0.25$ m

Width at outlet, $B_2 = 50$ mm $= 0.05$ m

Manometric efficiency, $\eta_{man} = 75\% = 0.75.$

Let vane angle at outlet $= \phi$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.25 \times 1450}{60} = 18.98 \text{ m/s}$$

Discharge is given by $Q = \pi D_2 B_2 \times V_{f_2}$

$$\therefore \quad V_{f_2} = \frac{Q}{\pi D_2 B_2} = \frac{0.118}{\pi \times 0.25 \times .05} = 3.0 \text{ r}$$

Using equation (19.8), $\eta_{man} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 25}{V_{w_2} \times 18.98}$

$$\therefore \quad V_{w_2} = \frac{9.81 \times 25}{\eta_{man} \times 18.98} = \frac{9.81 \times 25}{0.75 \times 18.98} = 17.23.$$

From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{(u_2 - V_{w_2})} = \frac{3.0}{(18.98 - 17.23)} = 1.7143$$

$$\therefore \quad \phi = \tan^{-1} 1.7143 = 59.74° \text{ or } \mathbf{59° \ 44'}. \textbf{ Ans.}$$

Problem (3) *A centrifugal pump delivers water against a net head of 14.5 metres and a design speed of 1000 r.p.m. The vanes are curved back to an angle of 30° with the periphery. The impeller diameter is 300 mm and outlet width is 50 mm. Determine the discharge of the pump if manometric efficiency is 95%.*

Solution. Given :

Net head, $H_m = 14.5$ m

Speed, $N = 1000$ r.p.m.

Vane angle at outlet, $\phi = 30°$

Impeller diameter means the diameter of the impeller at outlet

∴ Diameter, $D_2 = 300$ mm $= 0.30$ m

Outlet width, $B_2 = 50$ mm $= 0.05$ m

Manometric efficiency, $\eta_{man} = 95\% = 0.95$

Tangential velocity of impeller at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.30 \times 1000}{60} = 15.70 \text{ m/s.}$$

Now using equation (19.8), $\eta_{man} = \frac{gH_m}{V_{w_2} \times u_2}$

$$\therefore \quad 0.95 = \frac{9.81 \times 14.5}{V_{w_2} \times 15.70}$$

$$\therefore \quad V_{w_2} = \frac{0.95 \times 14.5}{0.95 \times 15.70} = 9.54 \text{ m/s.}$$

Refer to Fig. 19.5. From outlet velocity triangle, we have

$$\tan\phi = \frac{V_{f_2}}{(u_2 - V_{w_2})} \text{ or } \tan 30° = \frac{V_{f_2}}{(15.70 - 9.54)} = \frac{V_{f_2}}{6.16}$$

$\therefore$ $V_{f_2} = 6.16 \times \tan 30° = 3.556$ m/s.

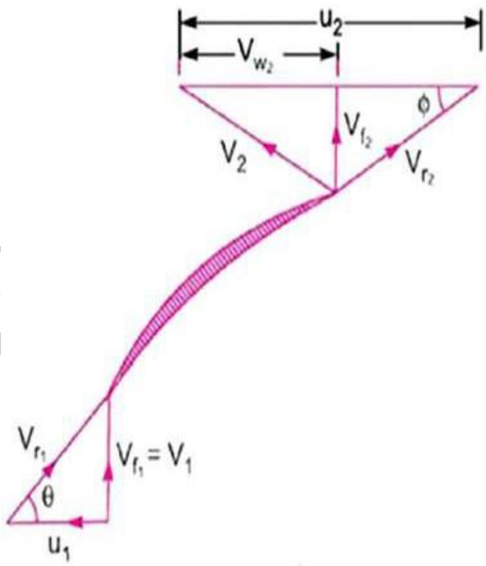
$\therefore$ Discharge, $Q = \pi D_2 B_2 \times V_{f_2}$

$= \pi \times 0.30 \times 0.05 \times 3.556 \text{ m}^3/\text{s} = \mathbf{0.1675\ m^3/s.}$ **Ans.**

Problem (4) *A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1000 r.p.m. works against a total head of 40 m. The velocity of flow through the impeller is constant and equal to 2.5 m/s. The vanes are set back at an angle of 40° at outlet. If the outer diameter of the impeller is 500 mm and width at outlet is 50 mm, determine :*

(*i*) *Vane angle at inlet,* (*ii*) *Work done by impeller on water per second, and*

(*iii*) *Manometric efficiency.*



Speed, $N = 1000$ r.p.m.

Head, $H_m = 40$ m

Velocity of flow, $V_{f_1} = V_{f_2} = 2.5$ m/s

Vane angle at outlet, $\phi = 40°$

Outer dia. of impeller, $D_2 = 500$ mm $= 0.50$ m

Inner dia. of impeller, $D_1 = \frac{D_2}{2} = \frac{0.50}{2} = 0.25$ m

Width at outlet, $B_2 = 50$ mm $= 0.05$ m

Tangential velocity of impeller at inlet and outlet are

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 1000}{60} = 13.09 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.50 \times 1000}{60} = 26.18 \text{ m/s.}$$

Discharge is given by, $Q = \pi D_2 B_2 \times V_{f_2} = \pi \times 0.50 \times .05 \times 2.5 = 0.1963 \text{ m}^3/\text{s}.$

(*i*) **Vane angle at inlet** **(θ).**

From inlet velocity triangle $\tan\theta = \frac{V_{f_1}}{u_1} = \frac{2.5}{13.09} = 0.191$

$\therefore$ $\theta = \tan^{-1} .191 = 10.81°$ or $\mathbf{10° 48'}$. **Ans.**

(*ii*) **Work done by impeller on water per second** is given by equation (19.2) as

$$= \frac{W}{g} \times V_{w_2} u_2 = \frac{\rho \times g \times Q}{g} \times V_{w_2} \times u_2$$

$$= \frac{1000 \times 9.81 \times 0.1963}{9.81} \times V_{w_2} \times 26.18 \qquad \ldots(i)$$

But from outlet velocity triangle, we have

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{2.5}{(26.18 - V_{w_2})}$$

$\therefore$ $26.18 - V_{w_2} = \frac{2.5}{\tan\phi} = \frac{2.5}{\tan 40°} = 2.979$

$\therefore$ $V_{w_2} = 26.18 - 2.979 = 23.2$ m/s.

Substituting this value of V_{w_2} in equation (*i*), we get the work done by impeller as

$$= \frac{1000 \times 9.81 \times 0.1963}{9.81} \times 23.2 \times 26.18$$

$$= \mathbf{119227.9\ Nm/s.} \quad \textbf{Ans.}$$

(*iii*) **Manometric efficiency (η_{man}).** Using equation (19.8), we have

$$\eta_{man} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 40}{23.2 \times 26.18} = 0.646 = \mathbf{64.4\%.\ Ans.}$$

RECIPROCATING PUMP

Introduction:

- It is a hydraulic machine which converts mechanical energy into hydraulic energy (pressure energy).
- It is a type of positive displacement pump.
- It is suitable where small amount of water is to be delivered at higher pressure.
- While working, it sucks water at low pressure into a cylinder containing a reciprocating piston. The piston exerts a thrust force on the water and increases its pressure.

Advantages:

- It can deliver the required flow rate very precisely.
- It gives a continuous rate of discharge.
- It can deliver fluid at very high pressure.
- It provides high suction lift.
- No priming is needed.

Disadvantages:

- It requires high maintenance
- It gives low flow rate i.e. it discharges low amount of water..
- These are heavy and bulky in size.
- It has high initial cost.

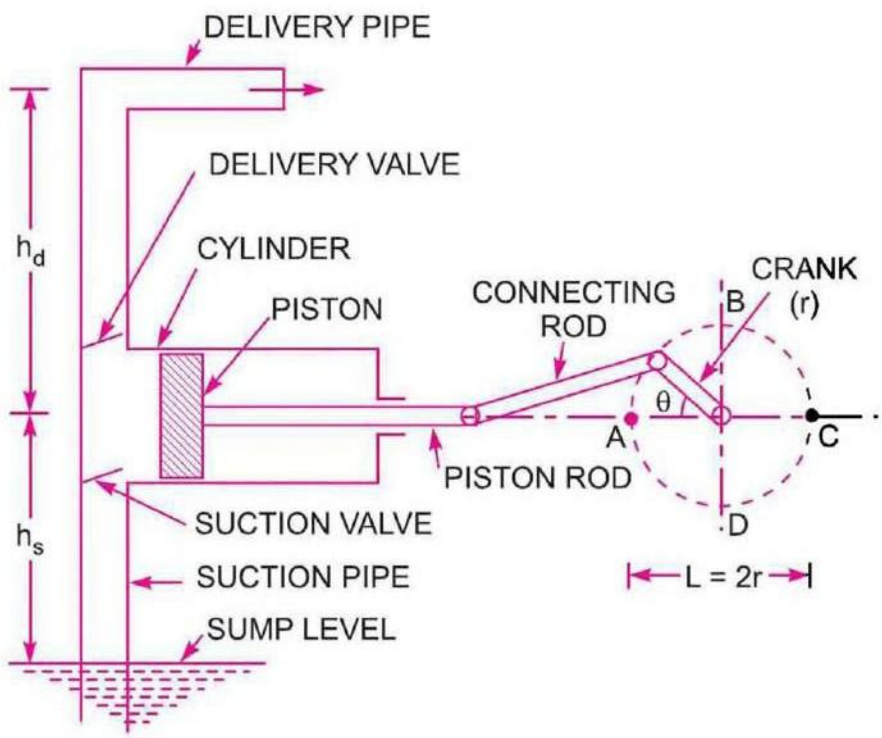
Classification of Reciprocating pump:

- According to sides in contact with water:
    - Single acting reciprocating pump
        - *In single acting reciprocating pump* water comes in contact of only one side of the piston. Suction and delivery of water occurs at one side.
    - Double acting reciprocating pump
        - *In double acting reciprocating pump* water comes in contact of both sides of the piston. Suction and delivery of water occurs at both sides.
- According to number of cylinders used:
    - Single cylinder pump
    - Double cylinder pump
    - Multi cylinder pump

Construction of Reciprocating pump:

Major components of Reciprocating pump are:

- A cylinder with piston, piston rod, connecting rod, crank and crank shaft
- Suction pipe
- Suction valve
- Delivery pipe
- Delivery valve



Working of Single acting reciprocating pump:

- The above figure shows the single acting reciprocating pump.
- It works in two strokes such as suction and delivery strokes.
- During suction stroke, the piston moves backward and suction valve opens. So, water enter into the cylinder. During suction the delivery valve remains closed.
- During delivery stroke, the piston moves forward and delivery valve opens. Suction valve remains closed. Piston exerts thrust on the water and increases water pressure.
- Water with pressure energy escapes out of the cylinder through delivery pipe to the delivery point.

Work done of Single acting reciprocating pump:

Consider the following terms:

D = diameter of the cylinder

A = cross-sectional area of the piston = $(\pi/4) \times D^2$

r = radius of crank

N =speed of crank in r.p.m

L = length of stroke = $\mathbf{2 \times r}$

H_s = suction head

H_d = delivery head

H = $\mathbf{H_s + H_d}$ = total head

Q = discharge of pump per second

ρ = density of water

Discharge of water in one revolution of crank = Volume of water delivered in one second = **A × L**

If, number of revolutions per sec = **N/60**

Discharge of pump per second, $\mathbf{Q = A \times L \times (N/60)}$

- Weight of water delivered /sec,

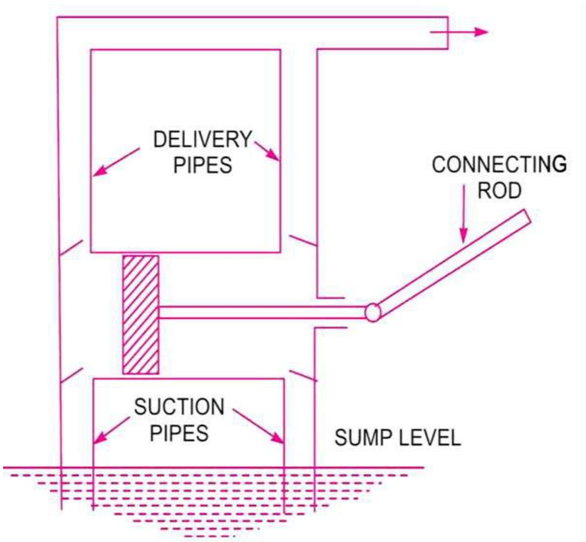
  $\mathbf{W = \rho \times g \times A \times L \times (N/60)}$

- Work done by the pump /sec,

  $\mathbf{W \times (H_s + H_d)}$

Working of Double acting reciprocating pump:

- When the piston moves right the suction valve of left side opens and suction valve of right side remains closed. The water is sucked into the cylinder at left side of piston.
- At this stroke delivery valve of left side remains closed and delivery valve of right side remains open. So, piston displaces the water with pressure energy at its right.
- Thus, suction occurs at left end of piston and discharge occurs at the right end.
- Similarly, when piston moves towards left, suction occurs at the right end and discharge occurs at the left end.



Work done of Double acting reciprocating pump:

Consider the following terms:

D = diameter of the cylinder

A = cross-sectional area of the piston = $(\pi/4) \times D^2$

r = radius of crank

N =speed of crank in r.p.m

L = length of stroke = **2 × r**

H_s = suction head

H_d = delivery head

H = $\mathbf{H_s + H_d}$ = total head

Q = discharge of pump per second

ρ = density of water

Discharge of water in one revolution of crank = Volume of water delivered in one second = 2×**A.L**

If, number of revolutions per sec = **N/60**

Discharge of pump per second, Q = 2×**A × L × (N/60)**

- Weight of water delivered /sec,

  $\mathbf{W = 2 \times \rho \times g \times A \times L \times (N/60)}$

- Work done by the pump /sec,

  $\mathbf{W \times (H_s + H_d) = 2 \times \rho \times g \times A \times L \times (N/60) \times (H_s + H_d)}$

Slip & Percentage of Slip:

Slip is the difference between the theoretical discharge (Q_{th}) and actual discharge (Q_{act}).

$\mathbf{Slip = Q_{th} - Q_{act}}$

This is known as *positive slip* when, Qth > Qact.

This is known as *negative slip* when, Qth < Qact

$$Percentage\ of\ slip = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100$$

Problem (1) ***A single acting reciprocating pump, running at 50 r.p.m delivers 0.01 m^3/s of water. The diameter of the piston is 200 mm and stroke length 400 mm. Determine: (i) the theoretical discharge of the pump, (ii) coefficient of discharge, (iii) slip and the percentage of slip of the pump.***

Solution: Given

Speed pf the pump, $N = 50$ r.p.m

Actual discharge, $Qa = 0.01\ m^3/s$

Diameter of piston, $D = 200$ mm $= 0.2$ m

Stroke length, $L = 400$ mm $= 0.4$ m

Cross-sectional area of piston, $A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times 0.2^2 = 0.031416\ m^2$

(i) Theoretical discharge of the pump, $Qth = \frac{A \times L \times N}{60} = \frac{0.031416 \times 0.4 \times 50}{60} = 0.01047\ m^3/s$

(ii) Coefficient of discharge, $C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.01}{0.01047} = 0.955$

(iii) slip of the pump, $Q_{th} - Q_{act} = 0.01047 - 0.01 = 0.00047\ m^3/s$

Percentage of slip of the pump $= \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100 = \frac{0.01047 - 0.01}{0.01047} \times 100 = 4.489\%$

Problem (2) A double acting reciprocating pump, running at 40 r.p.m delivers 1 m^3 of water per minute. The diameter of the piston is 200 mm and stroke length 400 mm. The delivery and suction head are 20 m and 5 m respectively. Find the slip of the pump and power required to drive the pump.

Solution: Given

Speed of the pump, $N = 40$ r.p.m

Actual discharge, $Q_{act} = 1 m^3/min = \frac{1}{60} m^3/s = 0.01666\ m^3/s$

Stroke length, $L = 400$ mm $= 0.4$ m

Diameter of piston, $D = 200$ mm $= 0.2$ m

Suction head, $H_s = 5$ m

Delivery head, $H_d = 20$ m

Cross-sectional area of piston, $A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times 0.2^2 = 0.031416\ m^2$

Theoretical discharge of the pump, $Qth = \frac{2 \times A \times L \times N}{60} = \frac{2 \times 0.031416 \times 0.4 \times 40}{60} = 0.01675\ m^3/s$

Slip of the pump, $Q_{th} - Q_{act} = 0.01675 - 0.01666 = 0.00009\ m^3/s$

Power required to drive the pump,

$$P = \frac{2 \times \rho \times g \times A \times L \times N \times (H_s + H_d)}{60000} = \frac{2 \times 1000 \times 9.81 \times 0.031416 \times 0.4 \times 40 \times (5 + 20)}{60000} = 4.109\ kW$$

Difference between Centrifugal pump and Reciprocating pump:

Centrifugal pump	Reciprocating pump
1. Simple in construction	1. Complicated in construction
2. Total weight of pump is less for a given discharge	2. Total weight of pump is more for a given discharge
3. Suitable for large discharge and smaller heads	3. Suitable for less discharge and higher heads
4. Required less floor area and simple foundation	4. Required more floor area and heavy foundation
5. Less wear and tear	5. More wear and tear
6. Maintenance cost is less	6. Maintenance cost is high
7. Can run at higher speeds	7. Can't run at higher speeds
8. Its delivery is continuous	8. Its delivery is pulsating
9.Needs priming	9. Doesn't need priming
10. It has less efficiency	10. It has more efficiency