CIRCUIT AND NETWORK THEORY

3RD Semester

Diploma in Electrical and Electronics Engineering

Prepared By:

Subash Chandra Behera

Department of Electrical and Electronics Engineering

Vedang Institute of Technology

Bhubaneswar, Khurda

CHAPTER 1

Circuit Elements and Laws

1.1 Voltage

Energy is required for the movement of charge from one point to another. Let W Joules of energy be required to move positive charge Q columbs from a point a to point b in a circuit. We say that a voltage exists between the two points. The voltage V between two points may be defined in terms of energy that would be required if a charge were transferred from one point to the other. Thus, there can be a voltage between two points even if no charge is actually moving from one to the other. Voltage between a and b is given by

$$V = \frac{W}{Q} J/C$$

$$\text{Hence Electric Potential (V)} = \frac{\text{Worked are (W) in Joules}}{\text{Ch arg e (Q)in columbs}}$$

Current :

An electric current is the movement of electric charges along a definite path. In case of a conductor the moving charges are electrons.

The unit of current is the ampere. The ampere is defined as that current which when flowing in two infinitely long parallel conductors of negligible cross section, situated 1 meter apart in Vacuum, produces between the conductors a force of 2×10^{-7} Newton per metre length.

Power : Power is defined as the work done per unit time. If a field F newton acts for t seconds through a distance d metres along a straight line, work done W = Fxd N.m. or J. The power P, either generated or dissipated by the circuit element.

$$P = \frac{w}{t} = \frac{F \times d}{t}$$

Power can also be written as Power = $\frac{\text{Work}}{\text{time}}$

$= \frac{\text{Work}}{\text{Charge}} \times \frac{\text{Charge}}{\text{Time}}$

= Voltage x Current P = V x I watt.

Energy : Electric energy W is defined as the Power Consumed in a given time. Hence, if current IA flows in an element over a time period t second, when a voltage V volts is applied across it, the energy consumed is given by

W = P x t = V x I x t J or watt. second.

The unit of energy W is Joule (J) or watt. second. However, in practice, the unit of energy is kilowatt. hour (Kwh)

1.2 **Resistance** : According to Ohm's law potential difference (V) across the ends of a conductor is proportional to the current (I) flowing through the conductor at a constant temperature. Mathematically Ohm's law is expressed as

$V \alpha I$ or $V = R \times I$

Or $R = \frac{V}{I}$ where R is the proportionality constant and is designated as the conductor resistance and has the unit of Ohm (Ω).

Conductance : Voltage is induced in a stationary conductor when placed in a varying magnetic field. The induced voltage (e) is proportional to the time rate of change of current, di/dt producing the magnetic field.

Therefore $e \alpha \frac{di}{dt}$

Or $e = L \frac{di}{dt}$

e and i are both function of time. The proportionality constant L is called inductance. The Unit of inductance is Henery (H).

Capacitance : A capacitor is a Physical device, which when polarized by an electric field by applying a suitable voltage across it, stores energy in the form of a charge separation.

The ability of the capacitor to store charge is measured in terms of capacitance. Capacitence of a capacitor is defined as the charge stored per Volt applied.

$$C = \frac{q}{v} = \frac{\text{Coulomb}}{\text{Volt}} = \text{Farad}$$

1.3 Active and passive Branch :

A branch is said to be active when it contains one or more energy sources. A passive branch does not contain an energy source.

Branch : A branch is an element of the network having only two terminals.

Bilateral and unilateral element :

A bilateral element conducts equally well in either direction. Resistors and inductors are examples of bilateral elements. When the current voltage relations are different for the two directions of current flow, the element is said to be unilateral. Diode is an unilateral element.

Linear Elements : When the current and voltage relationship in an element can be simulated by a linear equation either algebraic, differential or integral type, the element is said to be linear element.

Non Linear Elements : When the current and voltage relationship in an element can not be simulated by a linear equation, the element is said to be non linear elements.

1.4 Kirchhoff's Voltage Law (KVL) :

The algebraic sum of Voltages (or voltage drops) in any closed path or loop is Zero.

Application of KVL with series connected voltage source.

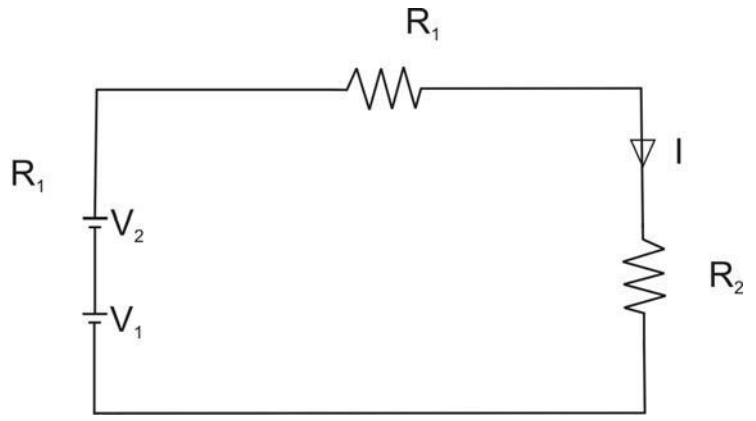


Fig. 1.1

$V_1+V_2-IR_1-IR_2=0$

$=V_1+V_2 =I (R_1+R_2)$

$$I = \frac{V_1 + V_2}{R_1 + R_2}$$

Application of KVL while voltage sources are connected in opposite polarity.

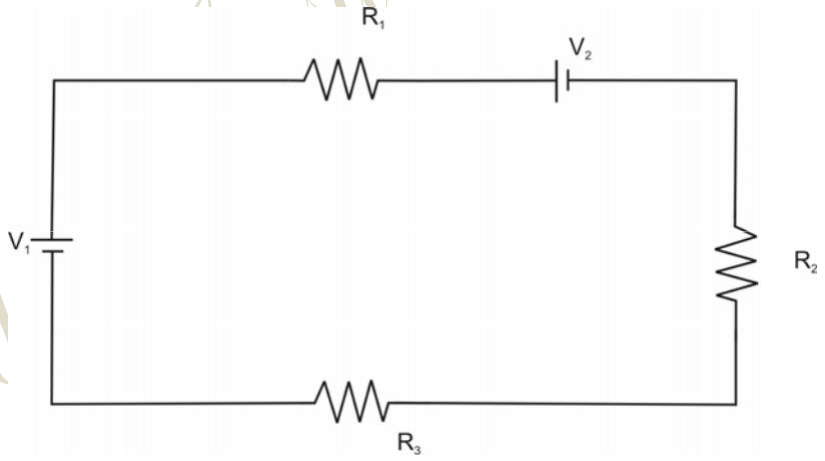


Fig. 1.2

$V_1 - IR_1 - V_2 - IR_2 - IR_3 = 0$

- $V_1 - V_2 = IR_1 + IR_2 + IR_3$
- $V_1 - V_2 = I (R_1 + IR_2 + IR_3)$

$$¾\ I = \frac{V_1 - V_2}{R_1 + R_2 + R_3}$$

Kirchaoff's Current Law (KCL) :

The algebraic sum of currents meeting at a junction or mode is zero.

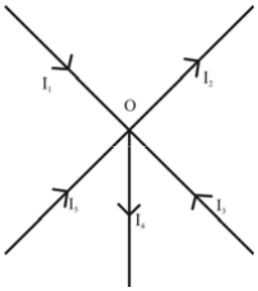


Fig. 1.3

Considering five conductors, carrying currents I_1, I_2, I_3, I_4 and I_5 meeting at a point O. Assuming the incoming currents to be positive and outgoing currents negative.

$$I_1 + (-I_2) + I_3 + (-I_4) + I_5 = 0$$

$$I_1 - I_2 + I_3 - I_4 + I_5 = 0$$

$$I_1 + I_3 + I_5 = I_2 + I_4$$

Thus above Law can also be stated as the sum of currents flowing towards any junction in an electric circuit is equal to the sum of the currents flowing away from that junction.

Voltage Division (Series Circuit)

Considering a voltage source (E) with resistors R_1 and R_2 in series across it.

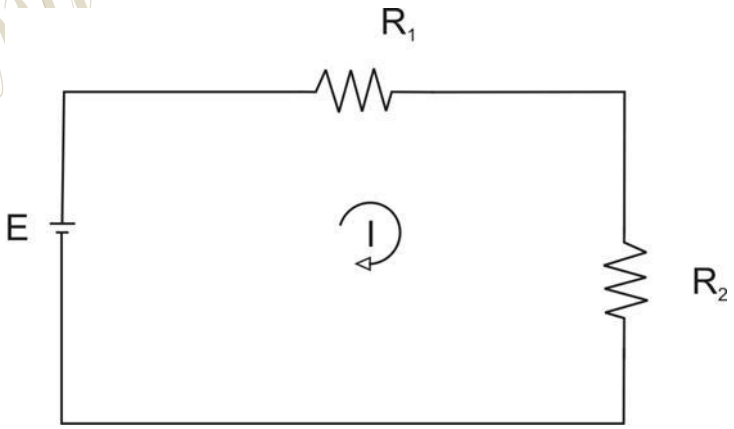


Fig. 1.4

$$I = \frac{E}{R_1+R_2}$$

$$\text{Voltage drop across } R_1 = I.R_1 = \frac{E.R_1}{R_1+R_2}$$

$$\text{Similarly voltage drop across } R_2 = I.R_2 = \frac{E.R_1}{R_1+R_2}$$

Current Division :

A parallel circuit acts as a current divider as the current divides in all branches in a parallel circuit.

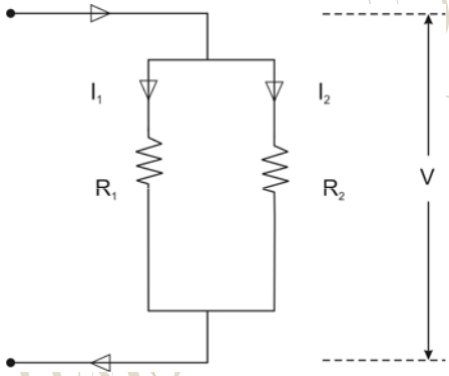


Fig. 1.5

Fig. shown the current I has been divided into I_1 and I_2 in two parallel branches with resistances R_1 and R_2 while V is the voltage drop across R_1 and R_2.

$$I_1 = \frac{V}{R_1} \text{ and } I_2 = \frac{V}{R_2}$$

Let R = Total resistance of the circuit.

$$\text{Hence } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\therefore R = \frac{R_1R_2}{R_1+R_2}$$

$$I = \frac{V}{R} = \frac{V}{\frac{R_1 R_2}{R_1+R_2}} = \frac{V(R_1 + R_2)}{R_1 R_2}$$

But = $V = I_1R_1 = I_2R_2$

¾ $\frac{R_1R_2}{R_1+R_2}$

¾ $I = \frac{I_1(R_1 + R_2)}{R_2}$

Therefore $\boxed{I_1 = \frac{IR_2}{R_1+R_2}}$

Similarly it can be derived that

$$\boxed{I_2 = \frac{IR_1}{R_1+R_2}}$$

CHAPTER 2

Magnetic Circuits :

Introduction : Magnetic flux lines always form closed loops. The closed path followed by the flux lines is called a magnetic circuit. Thus, a magnetic circuit provides a path for magnetic flux, just as an electric circuit provides a path for the flow of electric current. In general, the term magnetic circuit applies to any closed path in space, but in the analysis of electro-mechanical and electronic system this term is specifically used for circuits containing a major portion of ferromagnetic materials. The study of magnetic circuit concepts is essential in the design, analysis and application of electromagnetic devices like transformers, rotating machines, electromagnetic relays etc.

Magnetomotive Force (M.M.F) :

Flux is produced round any current – carrying coil. In order to produce the required flux density, the coil should have the correct number of turns. The product of the current and the number of turns is defined as the coil magneto motive force (m.m.f).

If I = Current through the coil (A)

N = Number of turns in the coil.

Magnetomotive force = Current x turns

So M.M.F = I X N

The unit of M.M.F. is ampere–turn (AT) but it is taken as Ampere(A) since N has no dimensions.

Magnetic Field Intensity

Magnetic Field Intensity is defined as the magneto-motive force per unit length of the magnetic flux path. Its symbol is H.

$$\text{Magnetic field Intensity (H)} = \frac{\text{Magnetomotive force}}{\text{Mean length of the magnetic path}}$$

¾ $H = \frac{F}{l} = \frac{I.N}{l}$ A/m

Where *l* is the mean length of the magnetic circuit in meters. Magnetic field intensity is also called magnetic field strength or magnetizing force.

Permeability :-

Every substance possesses a certain power of conducting magnetic lines of force. For example, iron is better conductor for magnetic lines of force than air (vaccum) . Permeability of a material (μ) is its conducting power for magnetic lines of force. It is the ratio of the flux density. (B) Produced in a material to the magnetic filed strength (H) i.e. $\mu = B / H$

Reluctance :

Reluctance (s) is akin to resistance (which limits the electric Current). Flux in a magnetic circuit is limited by reluctance. Thus reluctance(s) is a measure of the opposition offered by a magnetic circuit to the setting up of the flux.
Reluctance is the ratio of magneto motive force to the flux. Thus

$$S = Mmf / \varphi$$

Its unit is ampere turns per webber (or AT/wb)

Permeance:-

The reciprocal of reluctance is called the permeance (symbol A).

Permeance (A) = 1/S wb/AT

Turn T has no unit.

Hence permeance is expressed in wb/A or Henerys(H).

Electric Field versus Magentic Field.

Similarities

	Electric Field		Magnetic Field
1)	Flow of Current (I)	1)	Flow of flux (⌀)
2)	Emf is the cause of flow of current	2)	MMf is the cause of flow of flux
3)	Resistance offered to the flow of Current, is called resistance (R)	3)	Resistance offered to the flow of flux, is called reluctance (S)
4)	Conductance $\sigma = \frac{1}{R}$ ( )	4)	Permitivity (μ) = $1/S$
5)	Current density is amperes per square meter.	5)	Flux density is number of lines per square meter.
6)	Current (I) - EMF/R	6)	Flux (⌀) = $\frac{MMF}{S}$

Dissimilarities

1)	Current actually flows in an electric Circuit.	1)	Flux does not actually flow in a magnetic circuit.
2)	Energy is needed as long as current flows	2)	Energy is initially needed to create the magnetic flux, but not

			to maintain it.
3)	Conductance is constant and independent of current strength at a particular temperature.	3)	Permeability (or magnetic conductance) depends on the total flux for a particular temperature.

B.H. Curve :

Place a piece of an unmagnetised iron bar AB within the field of a solenoid to magnetise it. The field H produced by the solenoid, is called magnetising field, whose value can be altered (increased or decreased) by changing (increasing or decreasing) the current through the solenoid. If we increase slowly the value of magnetic field (H) from zero to maximum value, the value of flux density (B) varies along 1 to 2 as shown in the figure and the magnetic materials (i.e iron bar) finally attains the maximum value of flux density (Bm) at point 2 and thus becomes magnetically saturated.

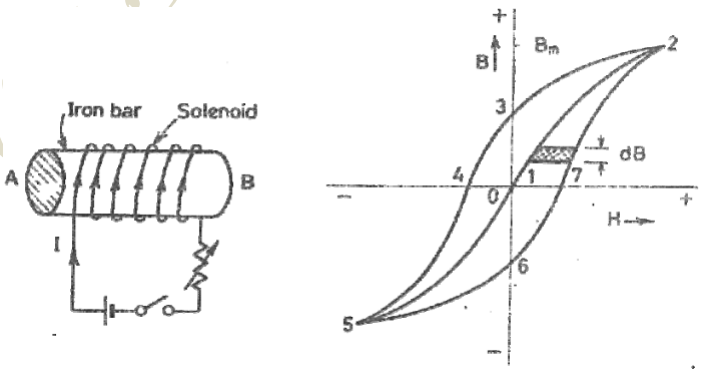


Fig. 2.1

Now if value of H is decreased slowly (by decreasing the current in the solenoid) the corresponding value of flux density (B) does not decreases along 2-1 but decreases some what less rapidly along 2 to 3. Consequently during the reversal of magnetization, the value of B is not zero, but is '13' at H= 0. In other

wards, during the period of removal of magnetization force (H), the iron bar is not completely demagnetized.

In order to demagnetise the iron bar completely, we have to supply the demagnetisastion force (H) in the opposite direction (i.e. by reserving the direction of current in the solenoid). The value of B is reduced to zero at point 4, when H='14'. This value of H required to clear off the residual magnetisation, is known as coercive force i.e. the tenacity with which the material holds to its magnetism.

If after obtaining zero value of magnetism, the value of H is made more negative, the iron bar again reaches, finally a state of magnetic saturation at the point 5, which represents negative saturation. Now if the value of H is increased from negative saturation (= '45') to positive saturation (= '12') a curve '5,6,7,2' is obtained. The closed loop "2,3,4,5,6,7,2" thus represents one complete cycle of magnetisation and is known as hysteresis loop.

NETWORK ANALYSIS

Different terms are defined below:

1. **Circuit:** A circuit is a closed conducting path through which an electric current either flow or is intended flow

2. **Network:** A combination of various electric elements, connected in any manner. Whatsoever, is called an electric network

3. **Node:** it is an equipotential point at which two or more circuit elements are joined.

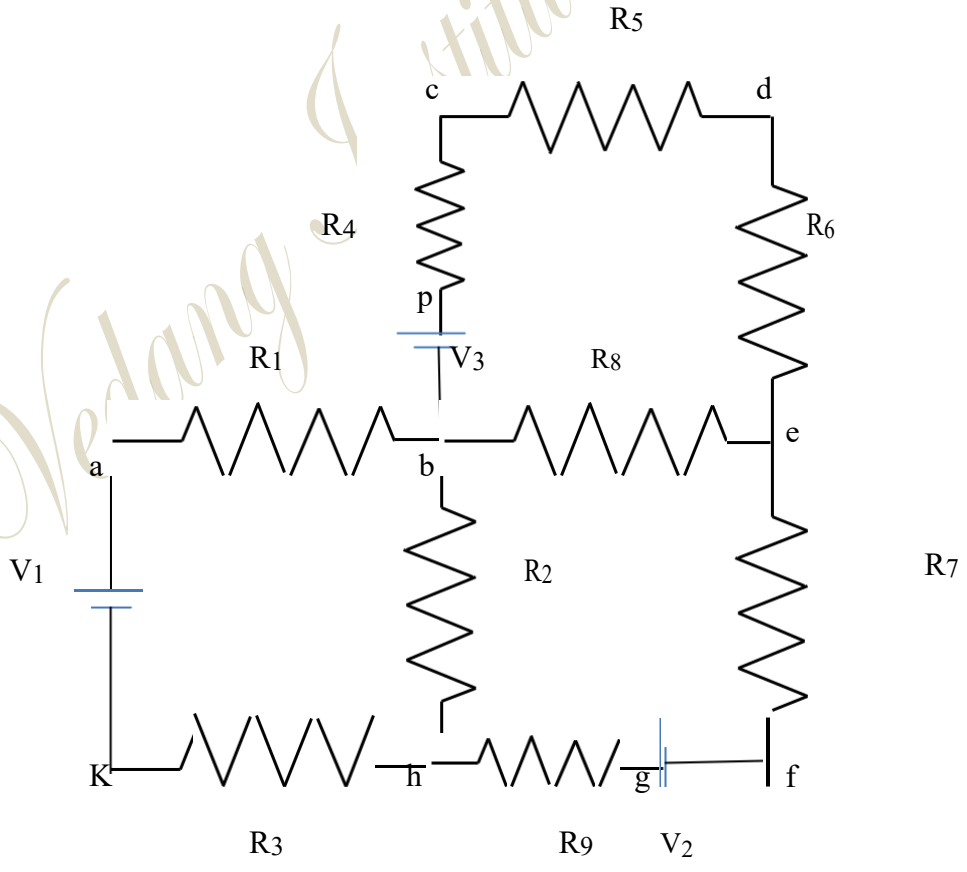
4. **Junction:** it is that point of a network where three or more circuit elements are joined.

5. **Branch:** it is a part of a network which lies between junction points.

6. **Loop:** It is a closed path in a circuit in which no element or node is accounted more than once.

7. **Mesh:** It is a loop that contains no other loop within it.

Example 3.1 In this circuit configuration of figure 3.1, obtain the no. of i) circuit elements ii) nodes iii) junction points iv) branches and v) meshes.



Solution: i) no. of circuit elements = 12 (9 resistors + 3 voltage sources)

ii) no. of nodes =10 (a, b, c, d, e, f, g, h, k, p)

iii) no. of junction points =3 (b, e, h)

iv) no. of branches = 5 (bcde, be, bh, befgh, bakh)

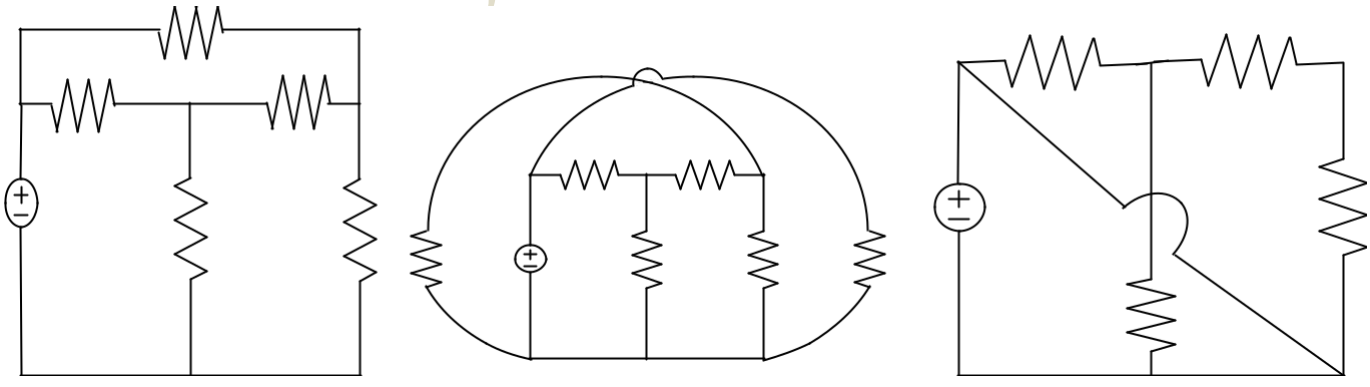
v) no. of meshes = 3 (abhk, bcde, befh)

3.2 MESH ANALYSIS

Mesh and nodal analysis are two basic important techniques used in finding solutions for a network. The suitability of either mesh or nodal analysis to a particular problem depends mainly on the number of voltage sources or current sources .If a network has a large number of voltage sources, it is useful to use mesh analysis; as this analysis requires that all the sources in a circuit be voltage sources. Therefore, if there are any current sources in a circuit they are to be converted into equivalent voltage sources,if, on the other hand, the network has more current sources,nodal analysis is more useful.

Mesh analysis is applicable only for planar networks. For non-planar circuits mesh analysis is not applicable .A circuit is said to be planar, if it can be drawn on a plane surface without crossovers. A non-planar circuit cannot be drawn on a plane surface without a crossover.

Figure 3.2 (a) is a planar circuit. Figure 3.2 (b) is a non-planar circuit and fig. 3.2 (c) is a planar circuit which looks like a non-planar circuit. It has already been discussed that a loop is a closed path. A mesh is defined as a loop which does not contain any other loops within it. To apply mesh analysis, our first step is to check whether the circuit is planar or not and the second is to select mesh currents. Finally, writing Kirchhoff's voltage law equations in terms of unknowns and solving them leads to the final solution.



(a) (b) (c)

Figure 3.2

Observation of the Fig.3.2 indicates that there are two loops abefa,and bcdeb in the network .Let us assume loop currents I_1 and $I_2$with directions as indicated in the figure.

Considering the loop abefa alone, we observe that current I_1 is passing through R_1, and (I_1-I_2) is passing through R_2. By applying Kirchhoff's voltage law, we can write

$$V_s = I_1R_1 + R_2(I_1 - I_2) \qquad (3.1)$$

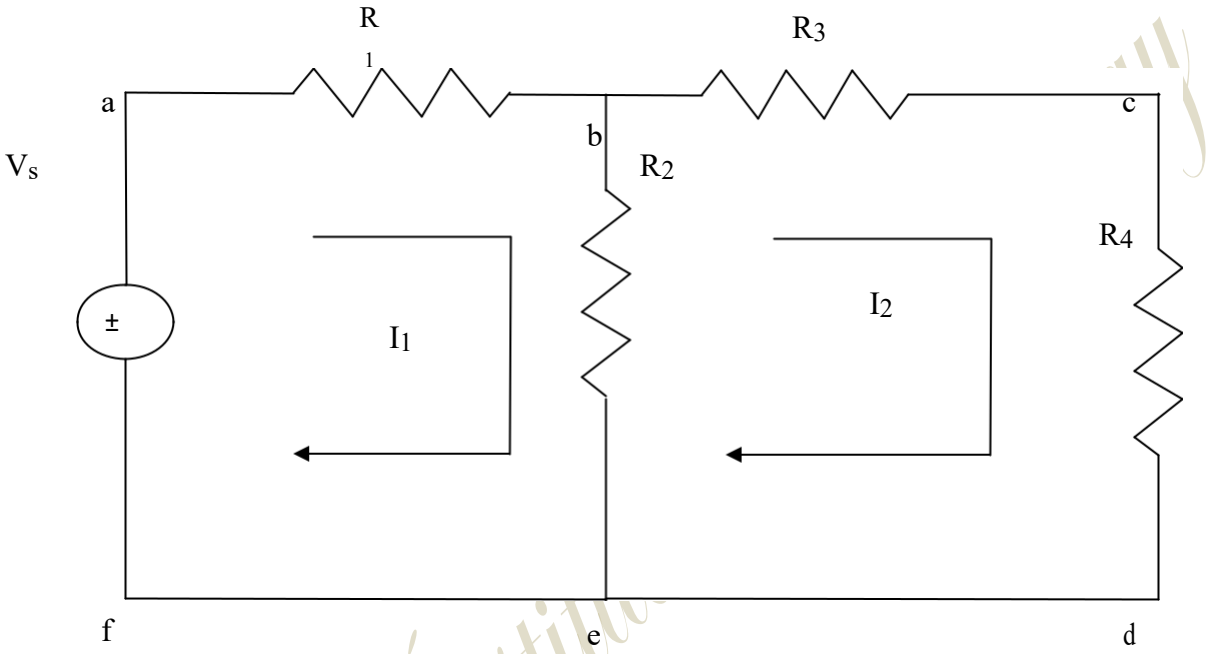


Figure 3.3

Similarly, if we consider the second mesh bcdeb, the current I_2 is passing through R_3 and R_4, and ($I_2 - I_1$) is passing through R_2. By applying Kirchhoff's voltage law around the second mesh, we have

$$R_2 (I_2 - I_1) + R_3I_2 + R_4I_2 = 0 \qquad (3.2)$$

By rearranging the above equations,the corresponding mesh current equations are

$$I_1 (R_1+R_2) - I_2R_2 = V_s.$$

$$-I_1R_2 + (R_2+R_3+R_4) I_2 = 0 \qquad (3.3)$$

By solving the above equations, we can find the currents I_1 and I_2,.If we observe Fig.3.3, the circuit consists of five branches and four nodes, including the reference node.The number of mesh currents is equal to the number of mesh equations.

And the number of equations=branches-(nodes-1).in Fig.3.3, the required number of

mesh current would be 5-(4-1)=2.

In general we have B number of branches and N number of nodes including the reference node than number of linearly independent mesh equations M=B-(N-1).

Example 3.2 Write the mesh current equations in the circuit shown in fig 3.4 and determine the currents.

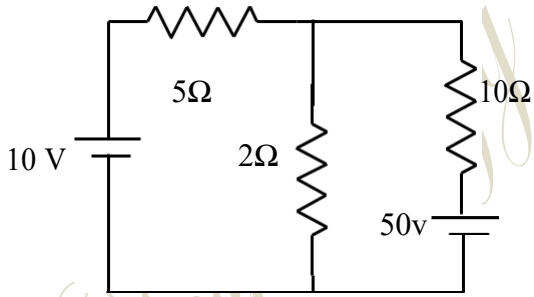


Figure 3.4

Solution: Assume two mesh currents in the direction as indicated in fig. 3.5. The mesh current equations are

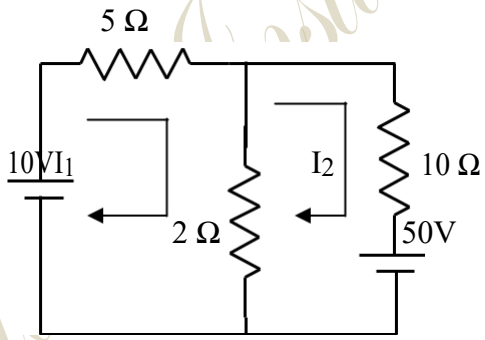


Figure 3.5

$5I_1 + 2(I_1-I_2) = 10$

$10I_2 + 2(I_2-I_1) + 50= 0$ (3.4)

We can rearrange the above equations as

$7I_1 -2I_2 =10$

$-2I_1+12I_2 = -50$ (3.5)

By solving the above equations, we have I_1= 0.25 A, and I_2 = -4.125

Here the current in the second mesh I_2, is negative; that is the actual current I_2 flows opposite to the assumed direction of current in the circuit of fig .3.5.

Example 3.3 Determine the mesh current I_1 in the circuit shown in fig.3.6.

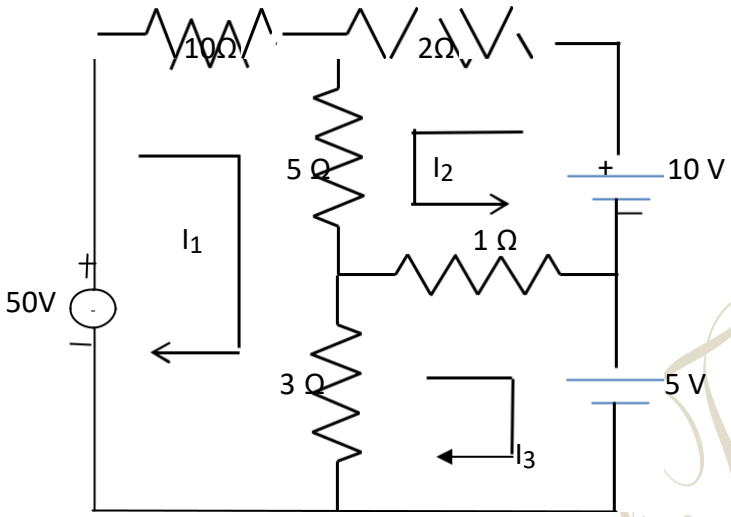


Figure 3.6

Solution: From the circuit, we can from the following three mesh equations

$$10I_1+5(I_1+I_2)+3(I_1-I_3) = 50 \quad (3.6)$$

$$2I_2+5(I_2+I_1)+1(I_2+I_3) = 10 \quad (3.7)$$

$$3(I_3-I_1)+1(I_3+I_2) = -5 \quad (3.8)$$

Rearranging the above equations we get

$$18I_1+5I_2-3I_3=50 \quad (3.9)$$

$$5I_1+8I_2+I_3=10 \quad (3.10)$$

$$-3I_1+I_2+4I_3=-5 \quad (3.11)$$

According to the Cramer's rule

$$I_1 = \frac{\begin{vmatrix} 50 & 5 & -3 \\ 10 & 8 & 1 \\ -5 & 1 & 4 \end{vmatrix}}{\begin{vmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{vmatrix}} = \frac{1175}{356}$$

Or $I_1 = 3.3$ A Similarly,

$$I_2 = \frac{\begin{vmatrix} 18 & 50 & -3 \\ 5 & 10 & 1 \\ -3 & -5 & 4 \end{vmatrix}}{\begin{vmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{vmatrix}} = \frac{-355}{356}$$

Or $I_2 = -0.997$A (3.12)

$$I_3 = \frac{\begin{vmatrix} 18 & 5 & 50 \\ 5 & 8 & 10 \\ -3 & 1 & -5 \end{vmatrix}}{\begin{vmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{vmatrix}} = \frac{525}{356}$$

Or $I_3 = 1.47$A (3.13)

$\therefore I_1 = 3.3$A, $I_2 = -0.997$A, $I_3 = 1.47$A

3.3 MESH EQUATIONS BY INSPECTION METHOD The mesh equations for a general planar network can be written by inspection without going through the detailed steps. Consider a three mesh networks as shown in figure 3.7

The loop equation are $I_1R_1 + R_2(I_1 - I_2) = V_1$

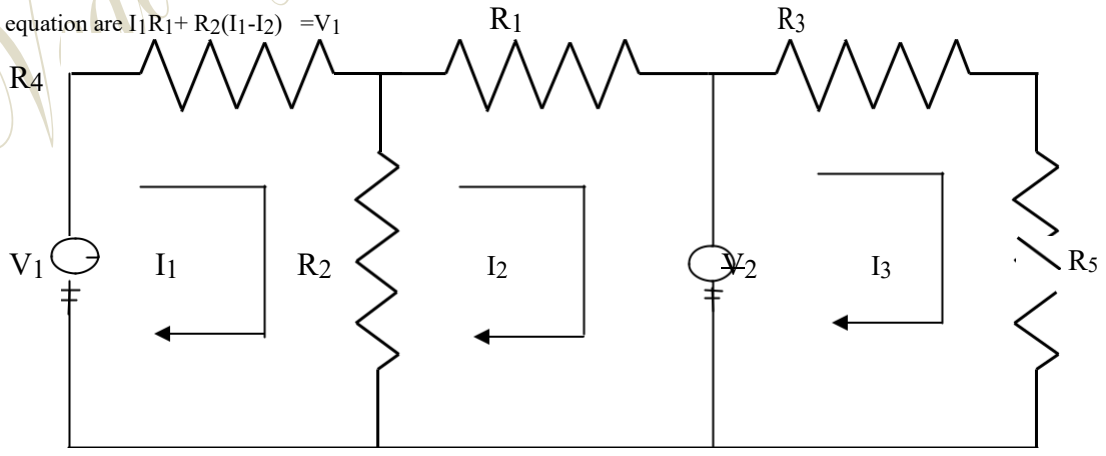


Figure 3.7

$R_2(I_2-I_1)+I_2R_3=-V_2$ 3.14

$R_4I_3+R_5I_3=V_2$ 3.15

Reordering the above equations, we have

$(R_1+R_2)I_1-R_2I_2=V_1$ 3.16

$-R_2I_1+(R_2+R_3)I_2=-V_2$ 3.17

$(R_4+R_5)I_3=V_2$ 3.18

The general mesh equations for three mesh resistive network can be written as

$R_{11}I_1 \pm R_{12}I_2 \pm R_{13}I_3= V_a$ 3.19

$\pm R_{21}I_1+R_{22}I_2 \pm R_{23}I_3= V_b$ 3.20

$\pm R_{31}I_1 \pm R_{32}I_2+R_{33}I_3= V_c$ 3.21

By comparing the equations 3.16, 3.17 and 3.18 with equations 3.19, 3.20 and 3.21 respectively, the following observations can be taken into account.

1. The self-resistance in each mesh
2. The mutual resistances between all pairs of meshes and
3. The algebraic sum of the voltages in each mesh.

The self-resistance of loop 1, $R_{11}=R_1+R_2$, is the sum of the resistances through which I_1 passes.

The mutual resistance of loop 1, $R_{12}= -R_2$, is the sum of the resistances common to loop currents I_1 and I_2. If the directions of the currents passing through the common resistances are the same, the mutual resistance will have a positive sign; and if the directions of the currents passing through the common resistance are opposite then the mutual resistance will have a negative sign.

$V_a=V_1$ is the voltage which drives the loop 1. Here the positive sign is used if the direction of the currents is the same as the direction of the source. If the current direction is opposite to the direction of the source, then the negative sign is used.

Similarly $R_{22}=R_2+R_3$ and $R_{33}=R_4+R_5$ are the self-resistances of loops 2 and 3 respectively. The mutual resistances $R_{13}=0$, $R_{21}= -R_2$, $R_{23}=0$, $R_{31}=0$, $R_{32}=0$ are the sums of the resistances common to the mesh currents indicated in their subscripts.

$V_b= -V_2$, $V_c= V_2$ are the sum of the voltages driving their respective loops.

Example 3.4 write the mesh equation for the circuit shown in fig. 3.8

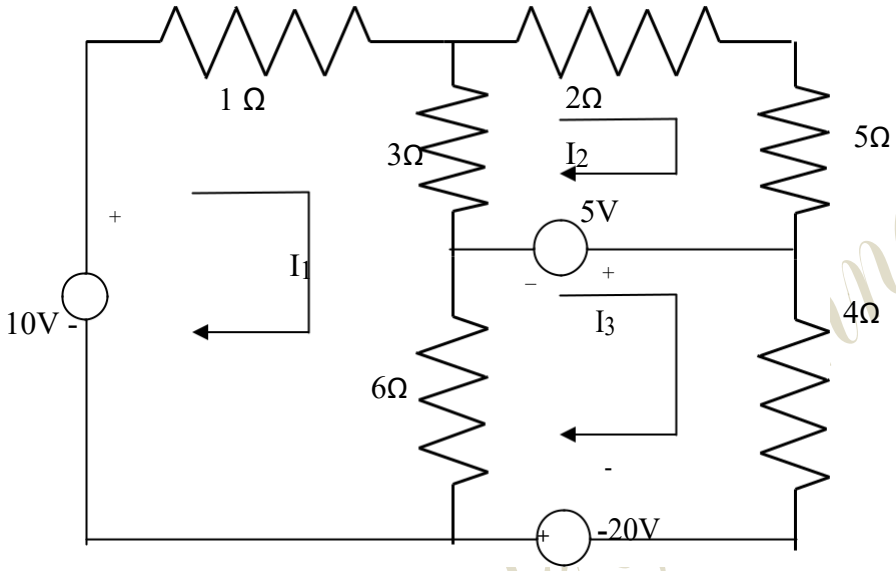


Figure 3.8

Solution : the general equation for three mesh equation are

$$R_{11}I_1 \pm R_{12}I_2 \pm R_{13}I_3 = V_a \quad (3.22)$$

$$\pm R_{21}I_1 + R_{22}I_2 \pm R_{23}I_3 = V_b \quad (3.23)$$

$$\pm R_{31}I_1 \pm R_{32}I_2 + R_{33}I_3 = V_c \quad (3.24)$$

Consider equation 3.22

R_{11}=self resistance of loop 1=(1Ω+ 3 Ω +6 Ω) =10 Ω

R_{12}= the mutual resistance common to loop 1 and loop 2 = -3 Ω

Here the negative sign indicates that the currents are in opposite direction

. R_{13}= the mutual resistance common to loop 1 & 3= -6 Ω

V_a= +10 V, the voltage the driving the loop 1.

Here he positive sign indicates the loop current I_1 is in the same direction as the source element.

Therefore equation 3.22 can be written as

$10\ I_1 - 3I_2 - 6I_3 = 10\ V$ (3.25)

Consider Eq. 3.23

R_{21}= the mutual resistance common to loop 1 and loop 2 = -3 Ω

R_{22}= self resistance of loop 2=(3Ω+ 2 Ω +5 Ω) =10 Ω

R_{23}=0, there is no common resistance between loop 2 and 3.

V_b = -5 V, the voltage driving the loop 2.

Therefore Eq. 3.23 can be written as

$-3I_1 + 10I_2 = -5V$ (3.26)

Consider Eq. 3.24

R_{31}= the mutual resistance common to loop 1 and loop 3 = -6 Ω

R_{32}= the mutual resistance common to loop 3 and loop 2 = 0

R_{33}= self resistance of loop 3=(6Ω+ 4 Ω) =10 Ω

V_c= the algebraic sum of the voltage driving loop 3

=(5 V+20V)=25 V (3.27)

Therefore, Eq3.24can be written as $-6I_1 + 10I_3 = 25V$

$-6I_1 - 3I_2 - 6I_3 = 10V$

$-3I_1 + 10I_2 = -5V$

$-6I_1 + 10I_3 = 25V$

3.4 SUPERMESH ANALYSIS

Suppose any of the branches in the network has a current source, then it is slightly difficult to apply mesh analysis straight forward because first we should assume an unknown voltage across the current source, writing mesh equation as before, and then relate the source current to the assigned mesh currents. This is generally a difficult approach. On way to overcome this difficulty is by applying the supermesh technique. Here we have to choose the kind of supermesh. A supermesh is constituted by two adjacent loops that have a common current source. As an example, consider the network shown in the figure 3.9.

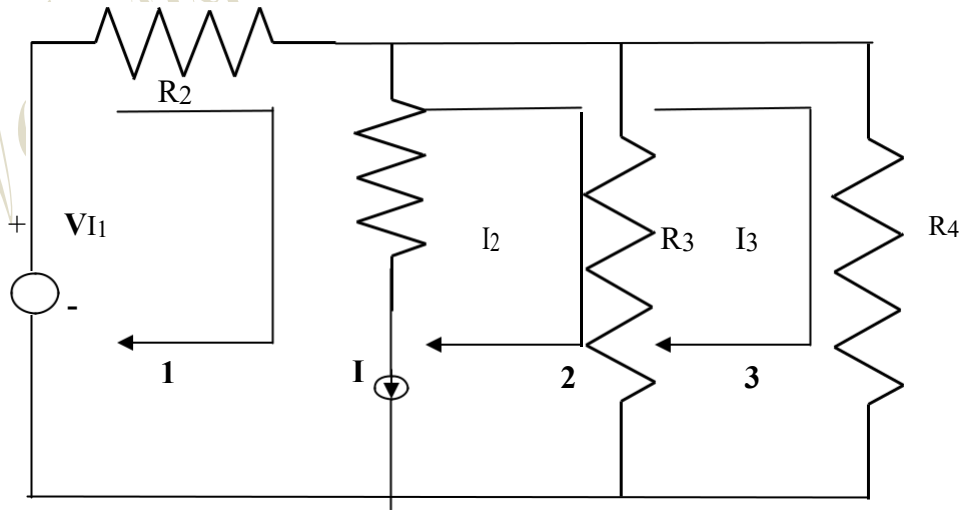


Figure 3.9

Here the current source I is in the common boundary for the two meshes 1 and 2. This current source creates a supermesh, which is nothing but a combination of meshes 1 and 2.

$$R_1I_1 + R_3(I_2-I_3)=V$$

Or $$R_1I_1 + R_3I_2 - R_4I_3= V$$

Considering mesh 3, we have

$$R_3(I_3-I_2)+ R_4I_3=0$$

Finally the current I from current source is equal to the difference between two mesh currents i.e.

$$I_1-I_2=I$$

we have thus formed three mesh equations which we can solve for the three unknown currents in the network.

Example 3.5. Determine the current in the 5Ω resistor in the network given in Fig. 3.10

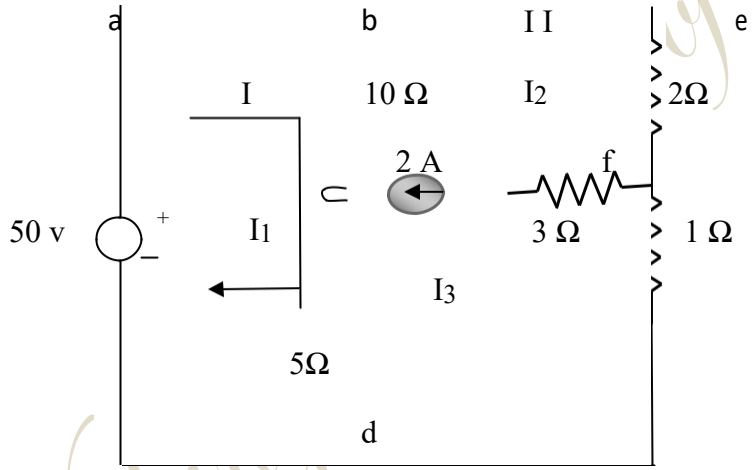


Figure 3.10

Solution: - From the first mesh, i.e. abcda, we have

$$50 = 10(I_1-I_2) + 5(I_1-I_3)$$

Or $$15I_1-10I_2 -5I_3 =50 \qquad (3.28)$$

From the second and third meshes. we can form a super mesh

$$10(I_2-I_1)+2I_2 +I_3+5(I_3-I_1)=0$$

Or $$-15I_1+12I_2 +6I_3 =0 \qquad (3.29)$$

The current source is equal to the difference between II and III mesh currents

i.e. $I_2-I_3 = 2A$ (3.30)

Solving 3.28.,3.29 and 3.30. we have

I_1 =19.99A,I_2= 17.33 A, and I_3 = 15.33

A The current in the 5Ω resistor =I_1 -I_3 =19.99

-15.33=4.66A

The current in the 5Ω resistor is 4.66A.

Example 3.6. Write the mesh equations for the circuit shown in fig. 3.11 and determine the currents, I_1, I_2 and I_3.

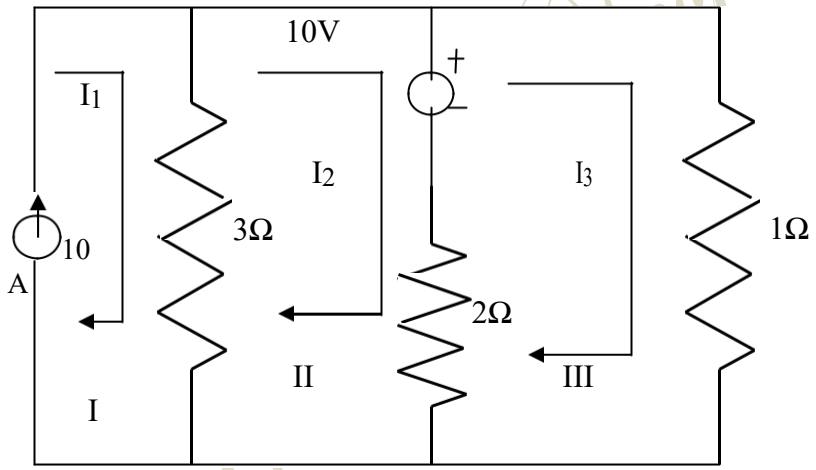


Figure 3.11

Solution ; In fig 3.11, the current source lies on the perimeter of the circuit, and the first mesh is ignored. Kirchhoff's voltage law is applied only for second and third meshes .

From the second mesh, we have

$3(I_2-I_1)+2(I_2-I_3)+10 =0$

Or $-3I_1 +5I_2-2I_3 = -10$ (3.31)

From the third mesh, we have

$I_3 + 2 (I_3 -I_2) =10$

Or $-2I_2+3I_3 =10$ (3.32)

From the first mesh, $I_1 = 10A$ (3.33)

From the abovethree equations, we get

$I_1=10A$, $I_2 =7.27$, $I_3 =8.18A$

3.5 NODALANALYSIS

In the chapter I we discussed simple circuits containing only two nodes, including the reference node. In general, in a N node circuit, one of the nodes is chosen as the reference or datum node, then it is possible to write N -1nodal equations by assuming N-1 node voltages. For example,a10 node circuit requires nine unknown voltages and nine equations. Each node in a circuit can be assigned a number or a letter. The node voltage is the voltage of a given node with respect to one particular node, called the reference node, which we assume at zero potential. In the circuit shown in fig. 3.12, node 3 is assumed as the Reference node. The voltage at node 1 is the voltage at that node with respect to node 3. Similarly, the voltage at node 2 is the voltage at that node with respect to node 3. Applying Kirchhoff's current law at node 1, the current entering is the current leaving (See Fig.3.13)

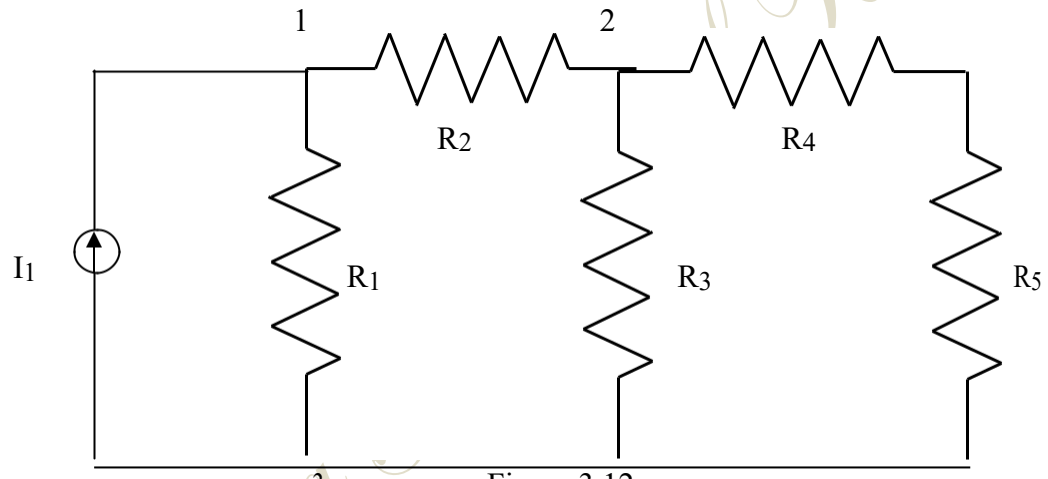


Figure 3.12

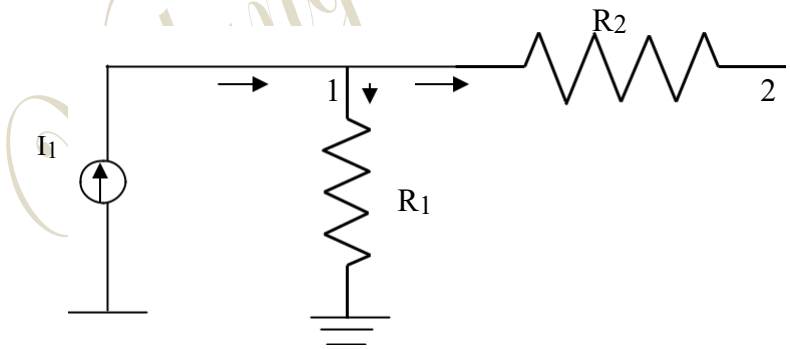


Figure 3.13

$I_1 = V_1/R_1 + (V_1-V_2)/R_2$

Where V_1 and V_2 are the voltages at node 1 and 2, respectively. Similarly, at node 2.the current entering is equal to the current leaving as shown in fig. 3.14

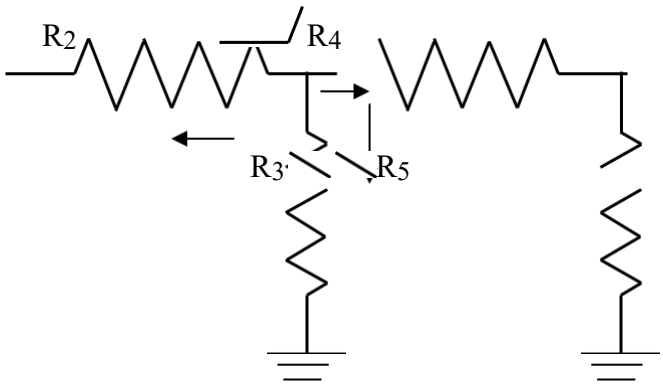


Figure 3.14

$(V_2-V_1)/R_2 + V_2/R_3 + V_2/(R_4+R_5) = 0$

Rearranging the above equations, we have

$V_1[1/R_1+1/R_2]-V_2(1/R_2)= I_1$

$-V_1(1/R_2) + V_2[1/R_2+1/R_3+1/(R_4+R_5)]=0$

From the above equations we can find the voltages at each node.

Example 3.7 Determine the voltages at each node for the circuit shown in fig 3.15

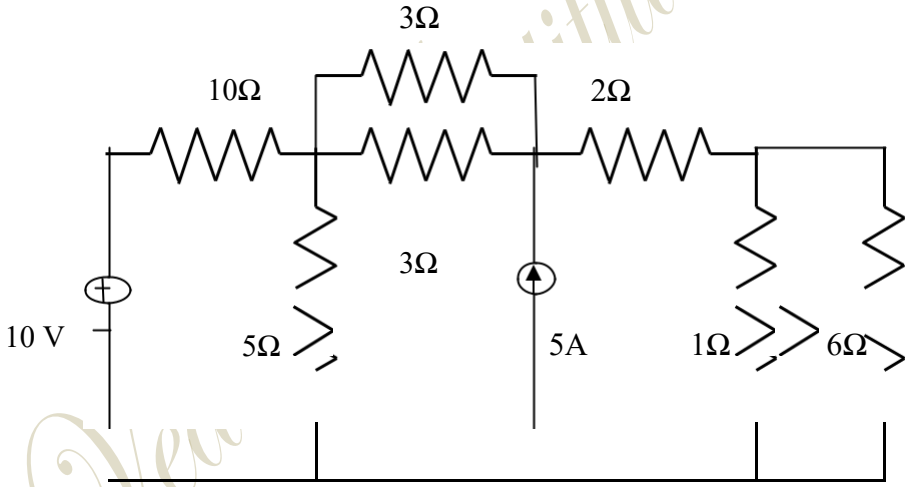


Figure 3.15

Solution : At node 1, assuming that all currents are leaving, we have

$(V_1-10)/10 + (V_1-V_2)/3 + V_1/5 + (V_1-V_2)/3 = 0$

Or $V_1[1/10 + 1/3 + 1/5 + 1/3] - V_2[1/3 + 1/3] = 1$

$0.96V_1-0.66V_2 = 1$ (3.36)

At node 2, assuming that all currents are leaving except the current from current source, we have

$(V_2-V_1)/3 + (V_2-V_1)/3 + (V_2-V_3)/2 = 5$

$-V_1[2/3]+V_2[1/3 + 1/3 + 1/2]-V_3(1/2) = 5$

$-0.66V_1+1.16V_2-0.5V_3= 5$ (3.37)

At node 3 assuming all currents are leaving, we

have $(V_3-V_2)/2 + V_3/1 + V_3/6 = 0$

$-0.5V_2 + 1.66V_3 = 0$ (3.38)

Applying Cramer's rule we get

$$V_1 = \frac{\begin{vmatrix} 1 & -0.66 & 0 \\ 5 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}} = \frac{7.154}{0.887} = 8.06$$

Similarly,

$$V_2 = \frac{\begin{vmatrix} 0.96 & 1 & 0 \\ -0.66 & 5 & -0.5 \\ 0 & 0 & 1.66 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}} = \frac{9.06}{0.887} = 10.2$$

$$V_3 = \frac{\begin{vmatrix} 0.96 & -0.66 & 1 \\ -0.66 & 1.16 & 5 \\ 0 & -0.5 & 0 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}} = \frac{2.73}{0.887} = 3.07$$

3.6 NODAL EQUATIONS BY INSPECTION METHOD The nodal equations for a general planar network can also be written by inspection without going through the detailed steps. Consider a three node resistive network, including the reference node, as shown in fig 3.16

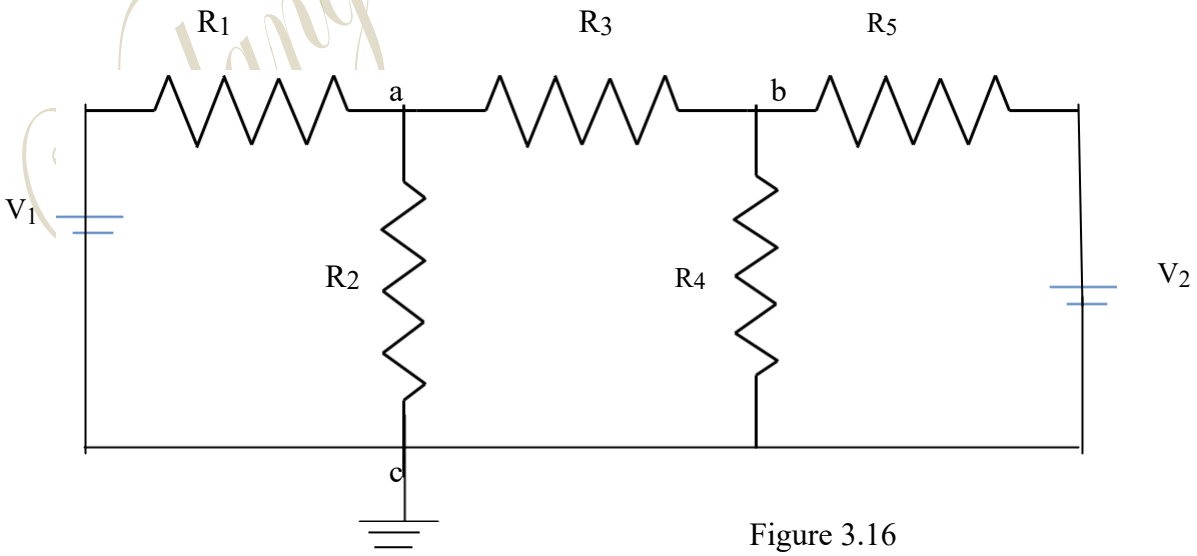


Figure 3.16

In fig. 3.16 the points a and b are the actual nodes and c is the reference node.

Now consider the nodes a and b separately as shown in fig 3.17(a) and (b)

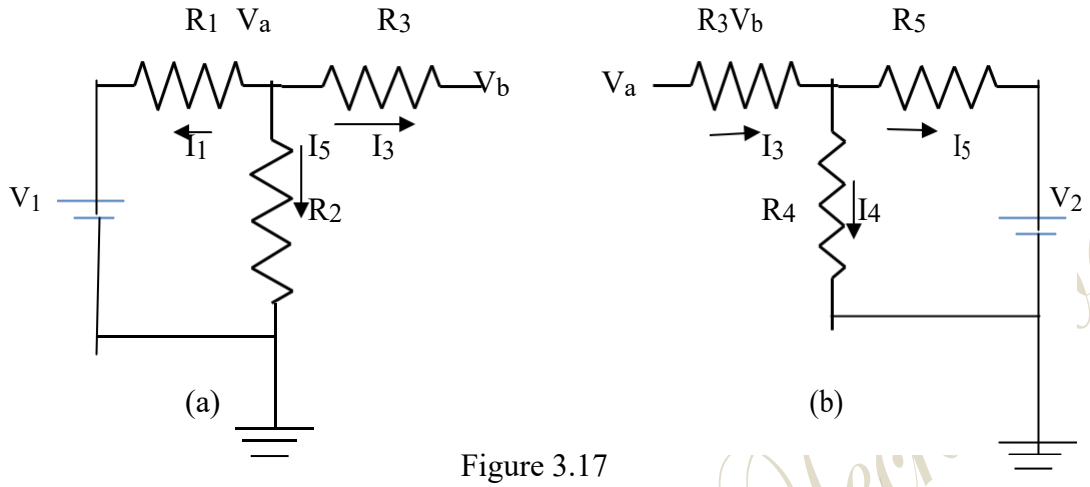


Figure 3.17

In fig 3.17 (a), according to Kirchhoff's current law we have

$I_1+I_2+I_3=0$

$(V_a-V_1)/R_1 +V_a/R_2+ (V_a-V_b)/R_3= 0$ (3.39)

In fig 3.17 (b) , if we apply Kirchhoff's current law

$I_4+ I_5= I_3$

$\therefore (V_b-V_a)/R_3 + V_b/R_4+(V_b-V_2)/R_5=0$ (3.40)

Rearranging the above equations we get

$(1/R_1+1/R_2+1/R_3)V_a-(1/R_3)V_b=(1/R_1)V_1$ (3.41)

$(-1/R_3)V_a+ (1/R_3+1/R_4+1/R_5)V_b=V_2/R_5$ (3.42)

In general, the above equation can be written as

$G_{aa}V_a + G_{ab}V_b=I_1$ (3.43)

$G_{ba}V_a + G_{bb}V_b=I_2$ (3.44)

By comparing Eqs 3.41,3.42 and Eqs 3.43, 3.44 we have the self conductance at node a, $G_{aa}=(1/R_1 + 1/R_2 + 1/R_3)$ is the sum of the conductances connected to node a. Similarly, $G_{bb} = (1/R_3 + 1/R_4 +1/R_5)$ is the sum of the conductances connected to node b. $G_{ab}=(- 1/R_3)$ is the sum of the mutual conductances connected to node *a* and node *b*. Here all the mutual conductances have negative signs. Similarly, $G_{ba}= (-1/R_3)$ is also a mutual conductance connected between nodes *b* and *a*. I_1 and I_2 are the sum of the source currents at node *a* and node *b*, respectively. The current which drives into the node has positive sign, while the current that drives away from the node has negative sign.

Example 3.8 for the circuit shown in the figure 3.18 write the node equations by the inspection method.

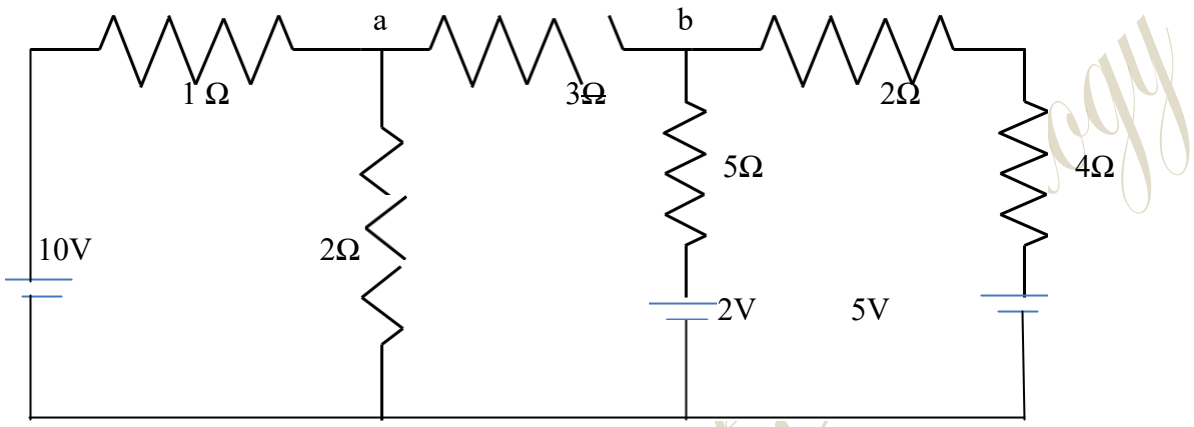


Fig 3.18

Solution:-

The general equations are

$G_{aa}V_a+G_{ab}V_b=I_1$ (3.45)

$G_{ba}V_a + G_{bb}V_b=I_2$ (3.46)

Consider equation 3.45

G_{aa}=(1+ 1/2 +1/3) mho. The self conductance at node *a* is the sum of the conductances connected to node *a*.

G_{bb} = (1/6 + 1/5 + 1/3) mho the self conductance at node *b* is the sum of conductances connected to node *b*.

G_{ab} =-(1/3) mho, the mutual conductances between nodes *a* and *b* is the sum of the conductances connected between node *a* and *b*.

Similarly G_{ba} = -(1/3), the sum of the mutual conductances between nodes *b* and *a*. I_1=10/1 =10 A, the source current at node *a*,

I_2=(2/5 + 5/6) = 1.23A, the source current at node *b*.

Therefore, the nodal equations are

$1.83V_a-0.33V_b=10$ (3.47)

$-0.33V_a+0.7V_b= 1.23$ (3.48)

3.7 SUPERNODE ANALYSIS

Suppose any of the branches in the network has a voltage source, then it is slightly difficult to apply nodal analysis. One way to overcome this difficulty is to apply the supernode technique. In this method, the two adjacent nodes that are connected by a voltage source are reduced to a single node and then the equations are formed by applying Kirchhoff's current law as usual. This is explained with the help of fig. 3.19

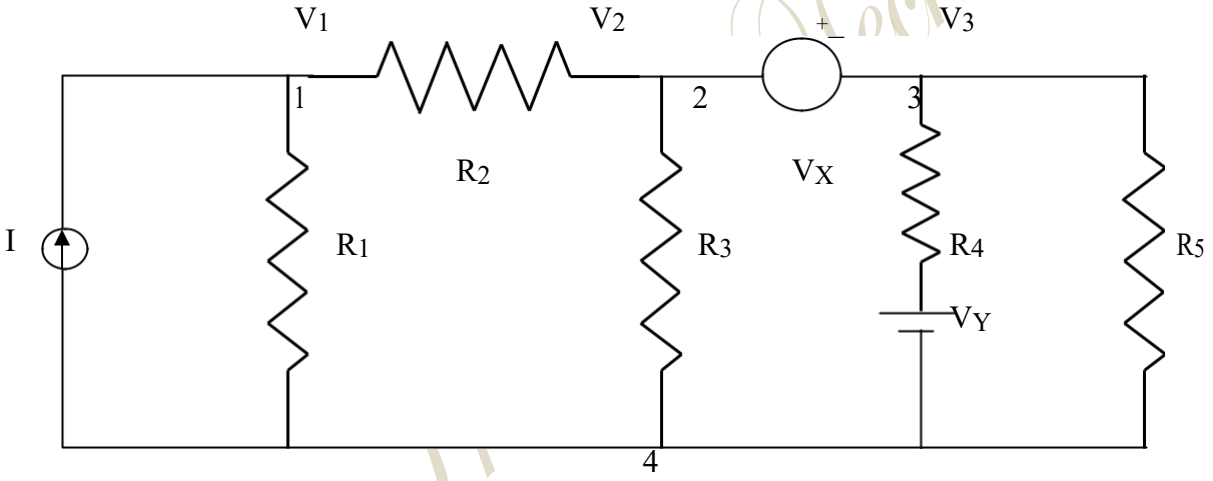


FIG 3.19

It is clear from the fig.3.19, that node 4 is the reference node. Applying Kirchhoff's current law at node 1, we get

$I=(V_1/R_1) + (V_1-V_2)/R_2$

Due to the presence of voltage source V_χ in between nodes 2 and 3 , it is slightly difficult to find out the current. The supernode technique can be conveniently applied in this case.

Accordingly, we can write the combined equation for nodes 2 and 3 as under.

$(V_2-V_1)/R_2 + V_2/R_3 + (V_3-V_y)/R_4 + V_3/R_5 = 0$

The other equation is

$V_2-V_3 = V_x$

From the above three equations, we can find the three unknown voltages.

Example 3.9 Determine the current in the 5 Ω resistor for the circuit shown in fig. 3.20

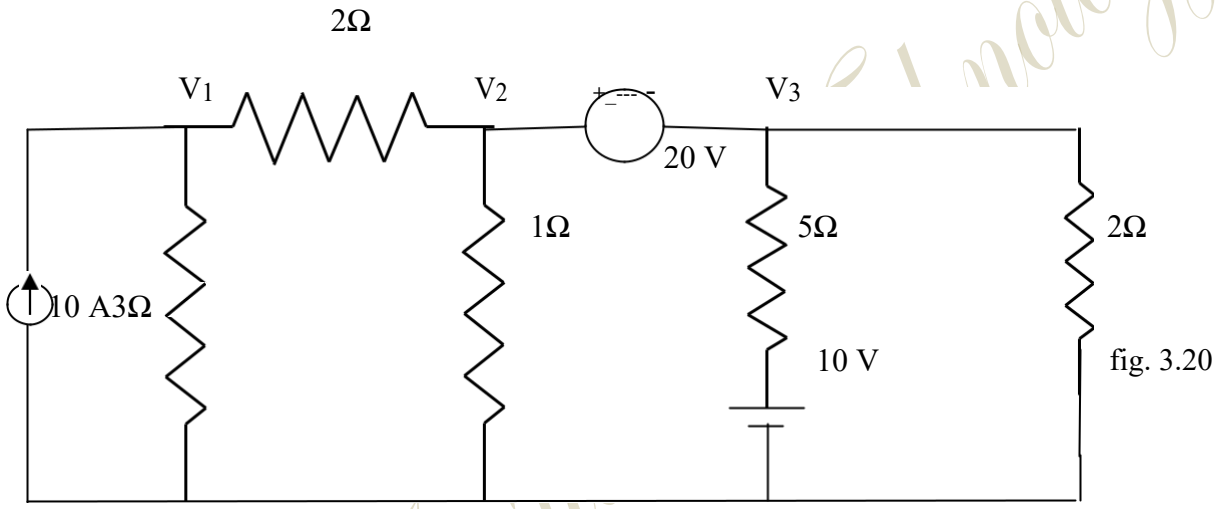


fig. 3.20

Solution. At node 1

$10 = V_1/3 + (V_1-V_2)/2$

Or $V_1[1/3 + 1/2]-(V_2/2)-10=0$

$0.83V_1-0.5V_2-10 = 0$ (3.49)

At node 2 and 3, the supernode equation is

$(V_2-V_1)/2 + V_2/1 + (V_3-10)/5 + V_3/2 = 0$

Or $-V_1/2 + V_2[(1/2)+1] + V_3[1/5 + 1/2]=2$

Or $-0.5V_1 + 1.5V_2+0.7V_3-2=0$ (2.50)

The voltage between nodes 2 and 3 is given by

$V_2-V_3=20$ (3.51)

The current in 5Ω resistor $I_5 = (V_3 - 10)/5$

Solving equation 3.49, 3.50 and 3.51, we obtain

$V_3 = -8.42$ V

∴ Currents $I_5 = (-8.42 - 10)/5 = -3.68$ A (current towards node 3) i.e the current flows towards node 3.

3.8 SOURCE TRANSFORMATION TECHNIQUE

In solving networks to find solutions one may have to deal with energy sources. It has already been discussed in chapter 1 that basically, energy sources are either voltage sources or current sources. Sometimes it is necessary to convert a voltage source to a current source or vice-versa. Any practical voltage source consists of an ideal voltage source in series with an internal resistance. Similarly, a practical current source consists of an ideal current source in parallel with an internal resistance as shown in figure3.21. R_v and R_i represent the internal resistances of the voltage source V_s , and current source I_s ,respectively.

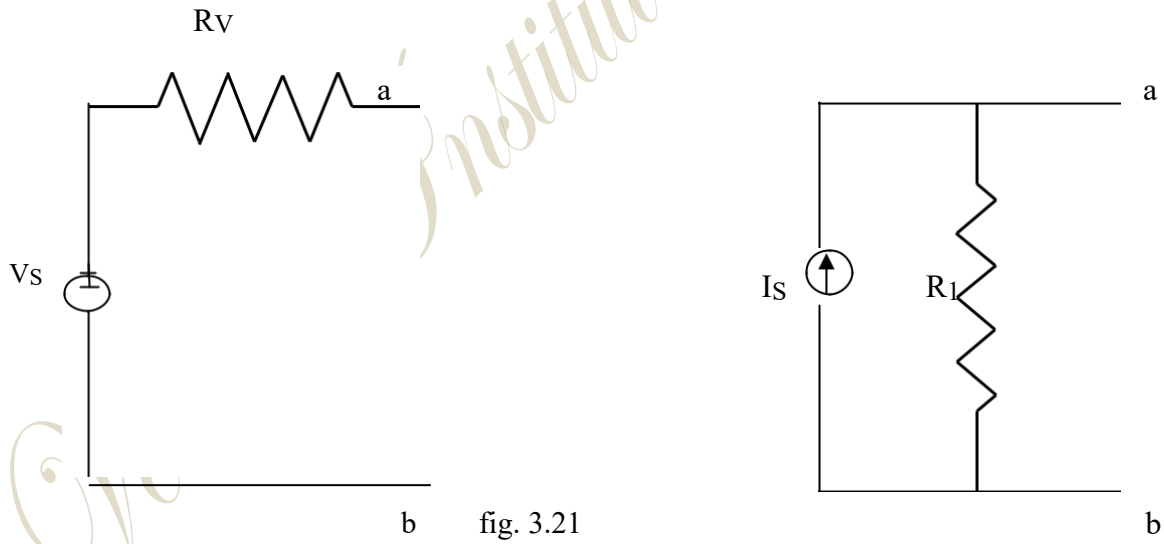


fig. 3.21

Any source, be it a current source or a voltage source, drives current through its load resistance, and the magnitude of the current depends on the value of the load resistance. Fig 3.22 represents a practical voltage source and a practical current source connected to the same load resistance R_L.

RV

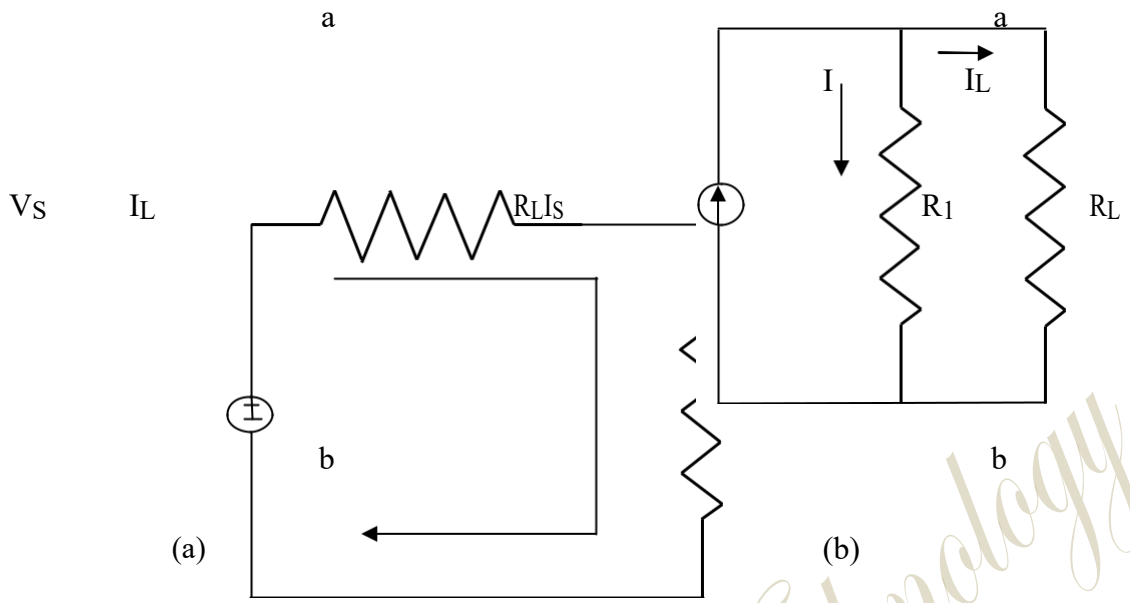


Figure 3.22

From fig 3.22 (a) the load voltage can be calculated by using Kirchhoff's voltage law as

$V_{ab}=V_s-I_LR_v$

The open circuit voltage $V_{oc}=V_s$

The short circuit current $I_{sc}=\frac{V_s}{R_v}$

from fig 3.22 (b)

$I_L=I_s-I=I_s-(V_{ab}/R_1)$

The open circuit voltage $V_{oc}= I_sR_1$

The short circuit current $I_{sc}=I_s$

The above two sources are said to be equal, if they produce equal amounts of current and voltage when they are connected to identical load resistances. Therefore, by equating the open circuit votages and short circuit currents of the above two sources we obtain

$V_{oc}=I_sR_1=V_s$

$I_{sc}=I_s=V_s/R_v$

It follows that

$R_1=R_v=R_s$; $V_s=I_sR_s$

where R_s is the internal resistance of the voltage or current source. Therefore, any practical voltage source, having an ideal voltage V_s and internal series resistance R_s can be replaced by a current source $I_s=V_s/R_s$ in parallel with an internal resistance R_s. The reverse

tansformation is also possible. Thus, a practical current source in parallel with an internal resistance R_S can be replaced by a voltage source $V_S=I_SR_S$ in series with an internal resistance R_S.

Example 3.10 Determine the equivalent voltage source for the current source shown in fig 3.23

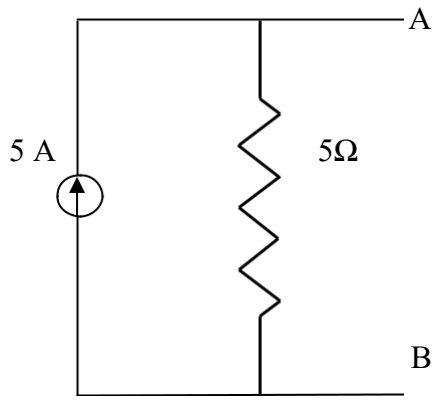


Figure 3.23

Solution: The voltage across terminals A and B is equal to 25 V. since the internal resistance for the current source is 5 Ω, the internal resistance of the voltage source is also 5 Ω. The equivalent voltage source is shown in fig. 3.24.

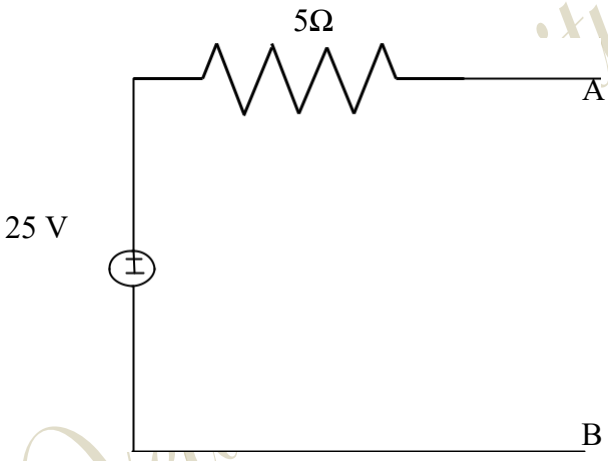
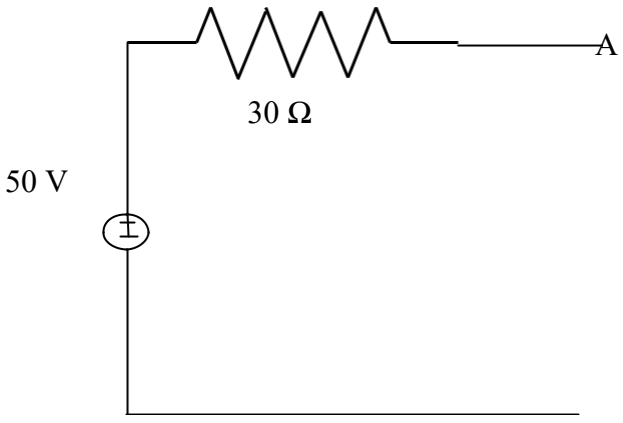


Fig 3.24

Example 3.11 Determine the equivalent current source for the voltage source shown in fig. 3.25



Solution : the short circuit current at terminals A and B is equal to

I= 50/30 = 1.66 A

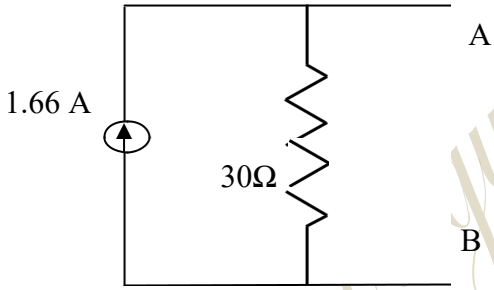


Fig 3.26

Since the internal resistance for the voltage source is 30Ω, the internal resistance of the current source is also 30 Ω. The equivalent current source is shown in fig. 3.26.

NETWORK THEOREMS

Before start the theorem we should know the basic terms of the network.

Circuit: It is the combination of electrical elements through which current passes is called circuit.

Network: It is the combination of circuits and elements is called network.

Unilateral :It is the circuit whose parameter and characteristics change with change in the direction of the supply application.

Bilateral: It is the circuit whose parameter and characteristics do not change with the supply in either side of the network.

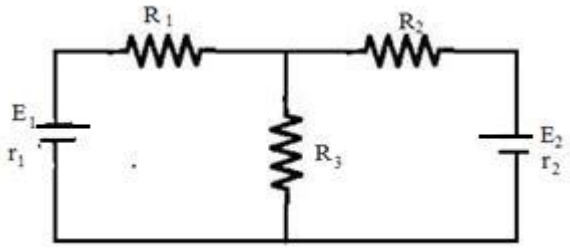
Node: It is the inter connection point of two or more than two elements is called node.

Branch: It is the interconnection point of three or more than three elements is called branch.

Loop: It is a complete closed path in a circuit and no element or node is taken more than once.

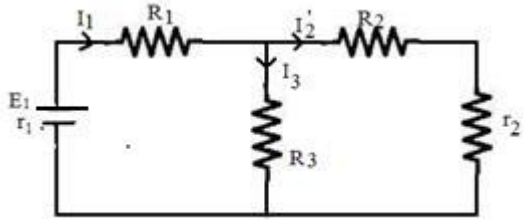
Super-Position Theorem :

Statement :" It states that in a network of linear resistances containing more than one source the current which flows at any point is the sum of all the currents which would flow at that point if each source were considered separately and all other sources replaced for time being leaving its internal resistances if any".



Explanation :

Considering E_1 source



Step 1.

R_2&r are in series and parallel with R_3 and again series with R_1

$(R_2+r_2) \parallel R_3$

$= \frac{(R_2 + r_2)R_3}{R_2 + r_2 + R_3} = m$ (say)

$Rt_1 = m + R_1 + r_1$

$I_1 = \frac{E_1}{Rt_1}$

$I_2 = \frac{I_1 \times R_3}{R_2 + r_2 + R_3}$

$I_3 = \frac{I_1 I_2 I_2}{R_2 + r_2 + R_3}$

Step – 2

Considering E2 source,R_1&r_2 are series and R_3 parallel and R_2 in series

$(R_1+r_1) \parallel R_3$

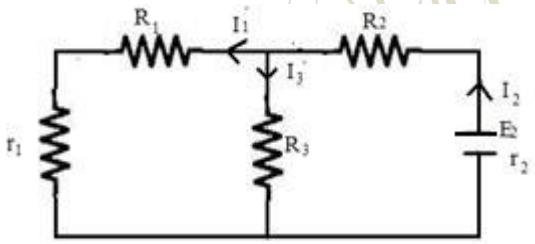
$= \frac{(R_1 + r_1)R_3}{R_1 + r_1 + R_3} = n$ (say)

$Rt_2 = n + R_2 + r_2$

$I_2 = \frac{E_2}{Rt_2}$

$I_3' = \frac{I_{21}(R_1 + r_1)}{R_1 + r_1 + R_3}$

$I_1' = \frac{I_2 \times R_3}{R_1 + r_1 + R_3}$



Step – 3

Current in R_1 branch = $I_1 - I_1'$

Current in R branch = $I_2 - I_2'$

The direction of the branch current will be in the direction of the greater value current.

Thevenin's Theorem :

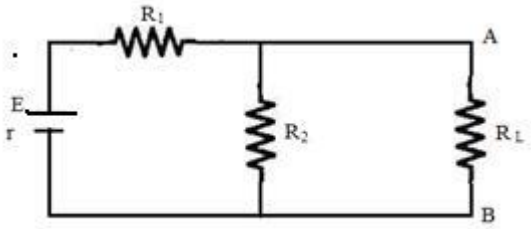
The current flowing through the load resistance R_1 connected across any two terminals A and B of a linear active bilateral network is given by

$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{V_{oc}}{R_i + R_L}$

Where $V_{th} = V_{oc}$ is the open. circuit voltage across A and B terminal when R_L is removed.

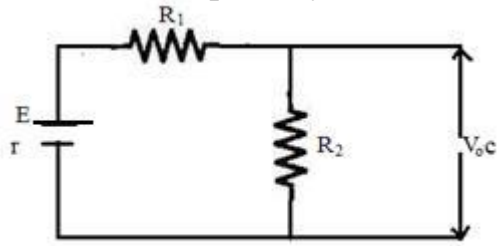
$R_i = R_{th}$ is the internal resistances of the network as viewed back into the open circuit network from terminals A & B with all sources replaced by their internal resistances if any.

Explanation :



<u>Step – 1</u> for finding V_{oc}

Remove R_L temporarily to find V_{oc}.

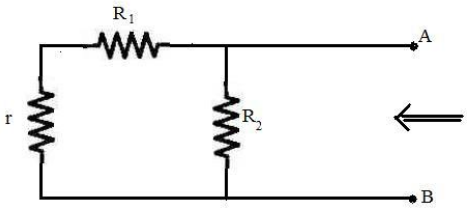


$$I = \frac{E}{R_1 + R_2 + r}$$

$$V_{oc} = IR_2$$

<u>Step – 2</u> finding R_{th}

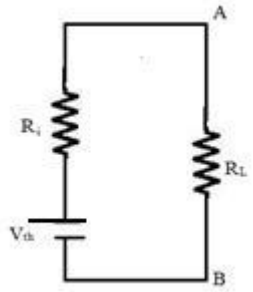
Remove all the sources leaving their internal resistances if any and viewed from open circuit side to find out R_i or R_{th}.



$$R_i = (R_1 + r) \parallel R_2$$

$$R_i = \frac{(R_1 + r)R_2}{R_1 + r + R_2}$$

<u>Step – 3</u>



Connect internal resistances and Thevenin's voltage in series with load resistance R_L.

Where R_{th}=thevenin resistance

V_{th}=thevenin voltage

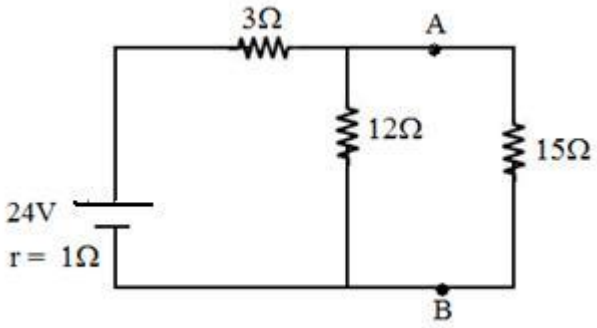
I_{th}=thevenin current

$R_i = (R_1 + r) \parallel R_2$

$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{V_{oc}}{R_i + R_L}$$

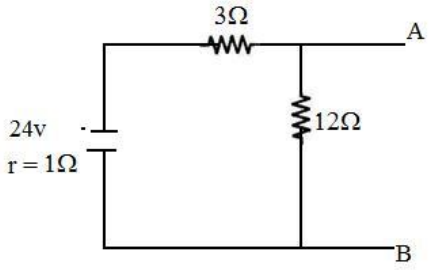
Example 01- Applying thevenin theorem find the following from given figure

(i) the Current in the load resistance R_L of 15 Ω



Solution : (i) Finding Voc

→ Remove 15Ω resistance and find the Voltage across A and B



V_{oc} is the voltage across 12 Ω resister

$$V_{oc} = \frac{24\times12}{12+3+1} = 18V$$

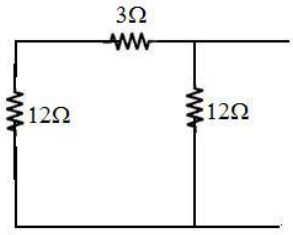
(ii) Finding R_{th}

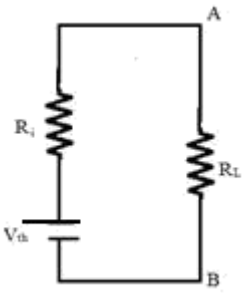
R_{th} is calculated from the terminal A & B into the network.

The 1 Ω resister and 3 Ω in are series and then parallel

$R_{th} = 3+1 \,//\, 12$

$$= \frac{4}{16}\times12 = 3\Omega$$





(iii) $I_{th} = = \frac{Voc}{R_L + R} = \frac{18}{15+3} = 1$ A.

Example 02: Determine the current in 1Ω resistor across AB of the network shown in fig(a) using thevenin theorem. **Solution:**The circuirt can be redrawn as in fig (b).

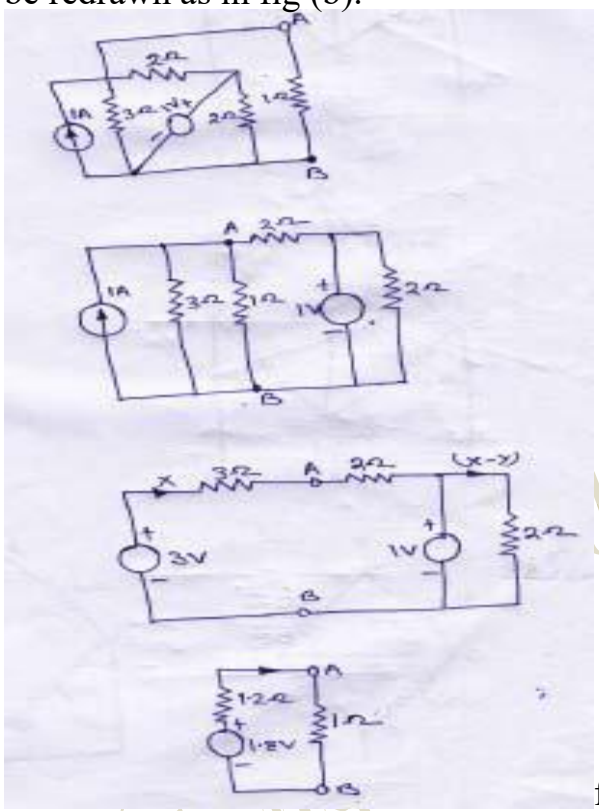


fig (a),(b),(c),(d) respectively

Step-1 remove the 1Ω resistor and keeping open circuit .The current source is converted to the equivalent voltage source as shown in fig (c)

Step-02 for finding the V_{th} we'll apply KVL law in fig (c)

then 3-(3+2)x-1=0

x=0.4A

$V_{th}=V_{AB}$ =3-3*0.4=1.8V

Step03-for finding the R_{th},all sources are set be zero

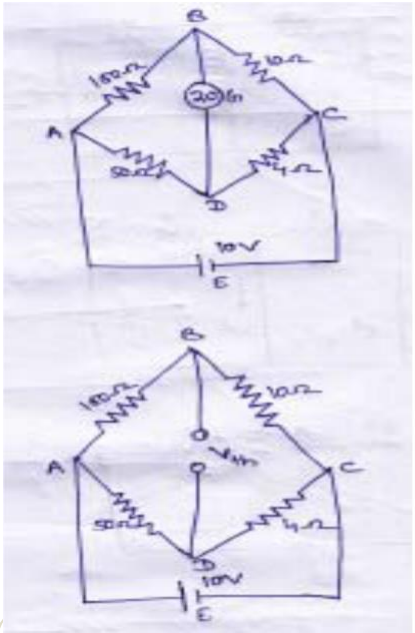
R_{th}=2//3=(2*3)/(2+3)=1.2Ω

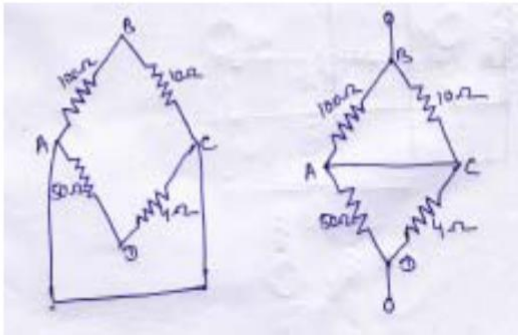
Step04- Then current I_{th}=1.8/(12.1+1)=0.82A

Example03: The four arms of a wheatstone bridge have the following resistances .

AB=100Ω,BC=10Ω,CD=4Ω,DA=50Ω.AA galvanometer of 20Ω resistance is connected across BD. Use thevenin theorem to compute the current through the galvanometer when the potential difference10V is maintained across AC.

Solution:





step 01- Galvanometer is removed.

step02-finding the V_{th} between B&D.ABC is a potential divider on which a voltage drop of 10vtakes place.

Potential of B w.r.t C=10*10/110=0.909V Potential of D w.r.t C=10*4/54=.741V then,

p.d between B&D is V_{th}=0.909-.741=0.168V Step03-finding R_{th}

remove all sources to zero keeping their internal resistances.

$R_{th} = R_{BD} = 10//100 + 50//4 = 12.79\Omega$

Step04;

lastly $I_{th} = V_{th}/R_{th} + R_L = 0.168/(12.79+20) = 5mA$

Norton's Theorem

Statement : In any two terminal active network containing voltage sources and resistances when viewed from its output terminals in equivalent to a constant current source and a parallel resistance. The constant current source is equal to the current which would flow in a short circuit placed across the terminals and parallel resistance is the resistance of the network when viewed from the open circuit side after replacing their internal resistances and removing all the sources.

OR

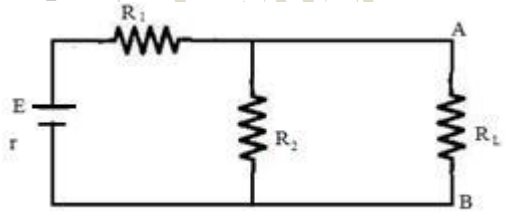
In any two terminal active network the current flowing through the load resistance R_L is given by

$$I = \frac{I_{sc} \times R_i}{R_i \times R_L}$$

Where R_i is the internal resistance of the network as viewed from the open ckt side A & B with all sources being replaced by leaving their internal resistances if any.

I_{sc} is the short ckt current between the two terminals of the load resistance when it is shorted

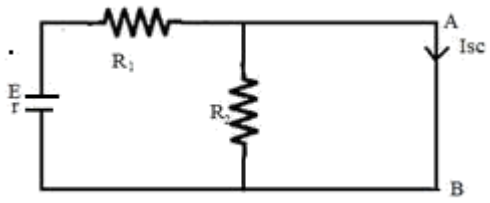
Explanation :



Step – 1

A &B are shorted by a thick copper wire to find out I_{sc}

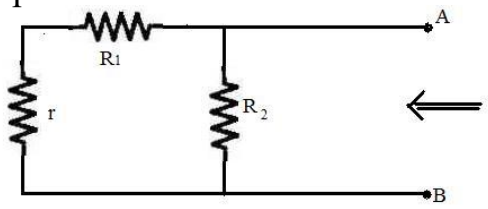
$I_{sc} = E/(R_1 + r)$



$I_{sc} = E / (R_1 + r)$

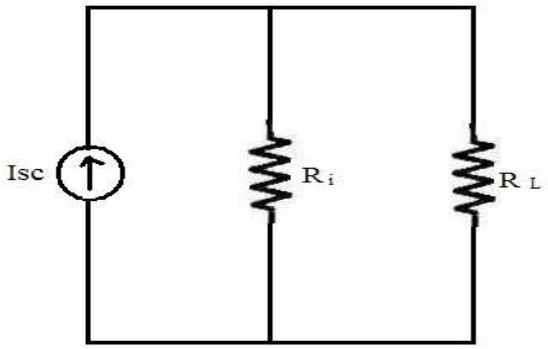
Step – 2

Remove all the source leaving its internal resistance if any and viewed from open circuit side A and B into the network to find R_i .



$R_i = (R_1 + r) \parallel R_2$

$R_i = (R_1 + r)R_2 /(R_1 + r + R_2)$



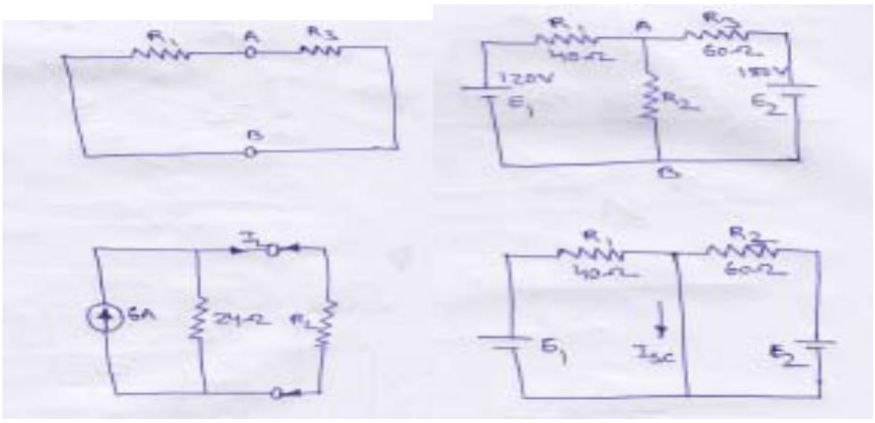
Step – 3

Connect I_{sc} & R_i in parallel with R_L

$$I = \frac{I_{sc} \times R_i}{R_i + R_L}$$

Example 01: Using norton's theorem find the current that would flow through the resistor R_2 whenit takes the values of 12Ω,24Ω&36Ω respectively in the fig shown below.

Solution:



Step 01-remove the load resistance by making short circuit. now terminal AB short circuited.

Step 02-Finding the short circuit current I_{sc}

First the current due to E_1 is =120/40=3A,and due to E_2 is 180/60=3A.

then I_{sc}=3+3=6A

Step 03-finding resistance R_N

It is calculated by by open circuit the load resistance and viewed from open circuit and into the network and all sources are taken zero.

R_N=40//60=(40*60)/(40+60)=24Ω

i) when R_L=12Ω, I_L=6*24/(24+36)=4A

ii) when R_L=24Ω,I_L=6/2=3A

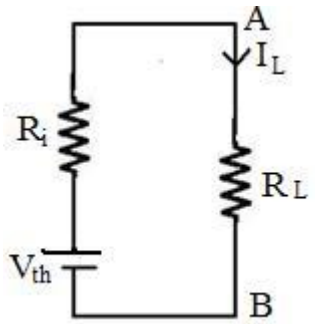
iii) when R_L=36Ω,I_L=6*24/(24+36)=2.4A

Maximum PowerTransfer Theorem

Statement : A resistive load will abstract maximum power from a network when the load resistance is equal to the resistance of the network as viewed from the output terminals(Open circuit) with all sources removed leaving their internal resistances if any

Proof :

$$I_L = \frac{V}{R_i + {}^{th}R_L}$$



Power delivered to the load resistance is given by

$$P_L = I_L^2 R_L$$

$$= \left(\frac{V_{th}}{R_i + R_L}\right)^2 R_L$$

$$= \frac{V_{th}^2 R_L}{(R_i + R_L)^2}$$

Power delivered to the load resistance R_L will be maximum

When $\frac{dP_L}{dR_L} = 0$

$$\Rightarrow \frac{d}{dR_L}\left[\frac{V_{th}^2 R_L}{(R_i + R_L)^2}\right] = 0$$

$$\Rightarrow \frac{V_{th}^2 (R_i + R_L)^2 - V_{th}^2 R_L \times 2(R_i + R_L)}{(R_i + R_L)^4} = 0$$

$$\Rightarrow V_{th}^2 (R_i + R_L)^2 - V_{th}^2 R_L \times 2(R_i + R_L) = 0$$

$$\Rightarrow V_{th}^2 (R_i + R_L)^2 - 2V_{th}^2 R_L (R_i + R_L) = 0$$

$$\Rightarrow V_{th}^2 (R_i + R_L)^2 = 2V_{th}^2 R_L (R_i + R_L)$$

$$\Rightarrow R_i + R_L = 2R_L$$

$$\Rightarrow R_i = 2R_L - R_L$$

$$\Rightarrow R_i = R_L$$

$$(P_L)\max = \frac{V_{th}^2}{(R_i + R_L)^2} R_L$$

$$= \frac{V_{th}^2}{4R_{2L}} R_L$$

$$= \frac{V_{th}^2}{4R_L^2} \times R_L$$

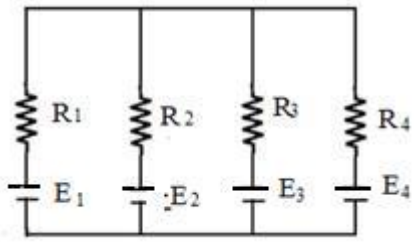
$$\boxed{(P_L)\max = \frac{V_{th}^2}{R_L}}$$

MILLIMAN'S THEOREM :

According to Millimans Theorem number of sources can be converted into a single source with a internal resistance connected in series to it,if the sources are in parallel connection.

According to the Milliman's theorem the equivalent voltage source

$$E_{eq} = \frac{E_1 \times \frac{1}{R_1} + E_2 \times \frac{1}{R_2} + E_3 \times \frac{1}{R_3} + ..}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...}$$

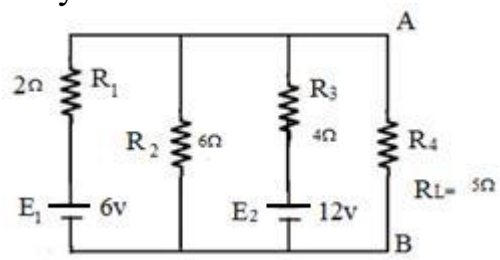


$$= \frac{E_1G_1 + E_2G_2 + E_3G_3 + \ldots}{G_1 + G_2 + G_3 + \ldots}$$

$$= \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2} + \frac{E_3}{R_3} + \ldots}{G_1 + G_2 + G_3 + \ldots}$$

$$= \frac{I_1 + I_2 + I_3 + \ldots}{G_1 + G_2 + G_3 + \ldots}$$

Example – Calculate the current across 5Ω resistor by using Milliman's Thm. Only



Solution :- Given ,

$R_1 = 2\Omega$, $R_2 = 6\Omega$, $R_3 = 4\Omega$, $R_L = 5\Omega$

$E_1 = 6v$, $E_2 = 12v$

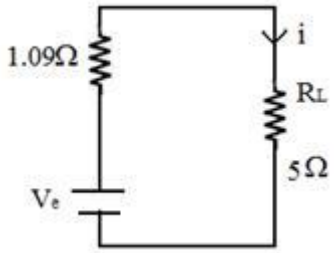
the resistance R_2 is not calculated because there is no voltage source

$$V_{ol} = E = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2} + \frac{E_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}.$$

$$= \frac{\frac{6}{2} + 0 + \frac{12}{4}}{\frac{1}{2} + \frac{1}{6} + \frac{1}{4}}.$$

$$= \frac{3+0+3}{\frac{6+2+3}{12}} \quad \frac{6}{11 \times 2} = 6.54v$$

$$R_1 = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{11}{12}} = \frac{12}{11} = 1.09.2$$



$$I_L = \frac{Voc}{1.09 + 5} = \frac{6.54}{1.09 + 5} = 1.07Amp.$$

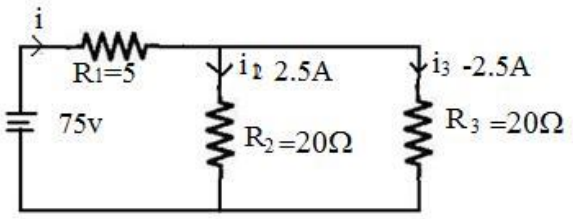
COMPENSATION THEOREM :

Statement :

It's states that in a circuit any resistance 'R" in a branch of network in which a current 'I' is flowing can be replaced. For the purposes of calculations by a voltage source = - IR

OR

If the resistance of any branch of network is changed from R to R +4R where the current flowing originaly is i. The change current at any other place in the network may be calculated by assuming that one e.m.f – I Δ R has been injected into the modified branch. While all other sources have their e.m.f. suppressed and 'R' represented by their internal resistances only.



Exp – (01)

Calculate the values of new currents in the network illustrated , when the resistor R_3 is increased by 30%.

Solution :- In the given circuit , the values of various branch currents are

$I_1 = 75/(5+10) = 5A$

$I_3 = I_2 = \frac{5 \times 20}{40} = 2.5\ Amp.$

Now the value of R_3, when it increase 30%

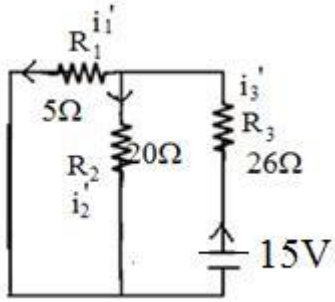
$R_3 = 20 + (20 \times 0.3) = 26\Omega$

$IR = 26 - 20 = 6\Omega$

$V = -I\Delta R$

$= -2.5 \times 6$

$= -15V$



$5 \parallel 20\Omega = \frac{5 \times 20}{5 + 20} = \frac{100}{25} =$

4Ω

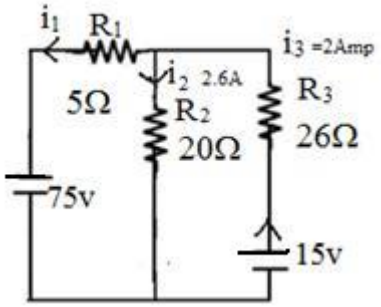
$I_3' = \frac{15}{4 + 26} = \frac{15}{30} = 0.5Amp$

$= 0.5 \times 5 =$

$I_2' = \frac{}{25} 0.1Amp$

$= \frac{0.5 \times 20}{25} =$

$I_1' \quad 0.4Amp$



$I_1'' = 5 - 0.4 = 4.6Amp$

$I_2'' = 0.1 + 2.5 = 2.6Amp$

$I_3'' = 2.5 - 0.5 = 2Amp$

RECIPROCITY THEOREM :

Statement :

It states that in any bilateral network, if a source of e.m.f 'E' in any branch produces a current 'I' any other branch. Then the same e.m.f 'E' acting in the second branch would produce the same current 'I' in the 1st branch.

Step – 1 First ammeter B reads the current in this branch due to the 36v source, the current is given by

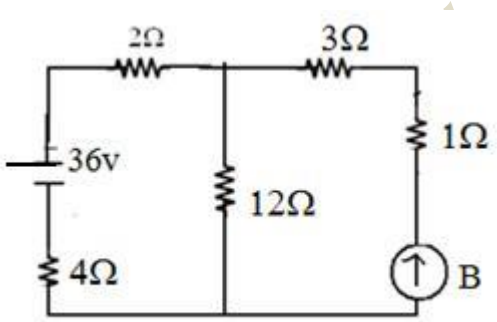
$4 \| 12 = \frac{4}{16} \times 12 = 3\Omega$

$R = 2+4+3=9\Omega$

$I = \frac{36}{9} = 4Amp$

$I_B = \frac{4\times12}{12+3+1} = \frac{48}{16} = 3Amp$

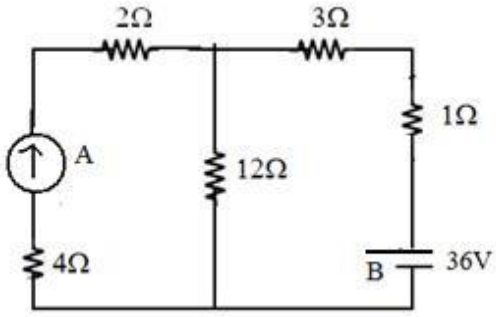
I_B =current through 1 Ω resister



Step – (II) Then interchanging the sources and measuring the current

$6\Omega \| 12\Omega = \frac{6\times12}{6} + 12 = \frac{72}{18} = 4\Omega$

$R = 4+3+1=8\Omega$



$I = \frac{36}{8} = 4.5Amp$, $I_A = \frac{4.5\times12}{6+2} = 3Amp$ Transfer resistance = $\frac{V}{I} = \frac{36}{3} = 12\Omega$.

COUPLED CIRCUITS

It is defined as the interconnected loops of an electric network through the magnetic circuit.

There are two types of induced emf.

(1) Statically Induced emf.

(2) Dynamically Induced emf.

Faraday's Laws of Electro-Magnetic :

Introduction → **First Law :→**

Whenever the magnetic flux linked with a circuit changes, an emf is induced in it.

OR

Whenever a conductor cuts magnetic flux an emf is induced in it.

Second Law :→

It states that the magnitude of induced emf is equal to the rate of change of flux linkages.

OR

The emf induced is directly proportional to the rate of change of flux and number of turns

Mathematically :

$$e \propto \frac{d\varphi}{dt}$$

$$e \propto N$$

Or $$e = -N \frac{d\varphi}{dt}$$

Where e = induced emf

N = No. of turns

ϕ = flux

'- ve' sign is due to Lenz's Law

Inductance :→

It is defined as the property of the substance which opposes any change in Current & flux.

Unit :→ Henry

Fleming's Right Hand Rule:→

It states that "hold your right hand with fore-finger, middle finger and thumb at right angles to each other. If the fore-finger represents the direction of field, thumb represents the direction of motion of the conductor, then the middle finger represents the direction of induced emf."

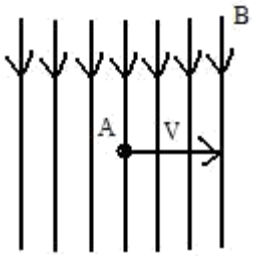
Lenz's Law : →

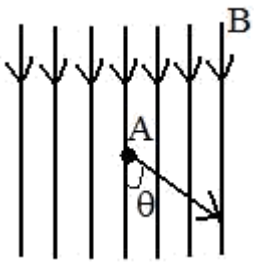
It states that electromagnetically induced current always flows in such a direction that the action of magnetic field set up by it tends to oppose the vary cause which produces it.

OR

It states that the direction of the induced current (emf) is such that it opposes the change of magnetic flux.

(2) Dynamically Induced emf :→





In this case the field is stationary and the conductors are rotating in an uniform magnetic field at flux density 'B" Wb/mt^2 and the conductor is lying perpendicular to the magnetic field. Let '*l*' is the length of the conductor and it moves a distance of 'dx' nt in time 'dt' second.

The area swept by the conductor = *l.*

dx Hence the flux cut = *ldx. B*

Change in flux in time 'dt' second = $\frac{Bldx}{dt}$

$E = \boldsymbol{Blv}$

Where $V = \frac{dx}{dt}$

If the conductor is making an angle 'θ' with the magnetic field, then

$e = Blv \sin\theta$

(1) Statically Induced emf :→

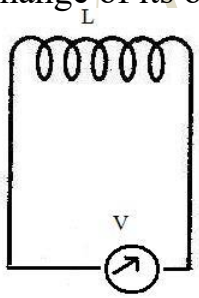
Here the conductors are remain in stationary and flux linked with it changes by increasing or decreasing.

It is divided into two types .

(i) Self-induced emf.

(ii) Mutually-induced emf.

(i) Self-induced emf : → It is defined as the emf induced in a coil due to the change of its own flux linked with the coil.



If current through the coil is changed then the flux linked with its own turn will also change which will produce an emf is called self-induced emf.

Self-Inductance :→

It is defined as the property of the coil due to which it opposes any change (increase or decrease) of current or flux through it.

Co-efficient of Self-Inductance (L) :→

It is defined as the ratio of weber turns per ampere of current in the coil.

OR

It is the ratio of flux linked per ampere of current in the coil

1st Method for 'L' :→

$$L = \frac{N\varphi}{I}$$

Where L = Co-efficient of self-induction

N = Number of turns

φ = flux

I = Current

2nd Method for L :→

We know that

$$L = \frac{N\varphi}{I}$$

$$\Rightarrow LI = N\varphi$$

$$\Rightarrow -LI = -N\varphi$$

$$\Rightarrow -L\frac{dI}{dt} = -N\frac{d\varphi}{dt}$$

$$\Rightarrow -L\frac{dI}{dt} = -N\frac{d\varphi}{dt}$$

$$\Rightarrow -L\frac{dI}{dt} = e$$

$$\Rightarrow L\frac{dI}{dt} = -e_L$$

$$\Rightarrow L = \frac{-e_L}{\frac{dI}{dt}}$$

Where L = Inductance

$e_L = -N\frac{d}{dt}\varphi$ is known as self-induced emf.

When $\frac{dI}{dt} = 1 amp/\text{ sec.}$

e = 1 volt

L = 1 Henry

A coil is said to be a self-inductance of 1 Henry if 1 volt is induced in it. When the current through it changes at the rate of 1 amp/ sec.

3rd Method for L :→

$$L = \frac{\mu_0 \mu_r AN^2}{l}$$

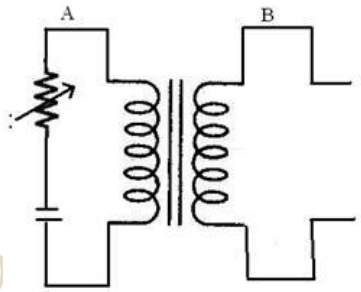
Where A = Area of x-section of the coil

N = Number of turns

L = Length of the coil

(ii) Mutually Induced emf :→

It is defined as the emf induced in one coil due to change in current in other coil. Consider two coils 'A' and 'B' lying close to each other. An emf will be induced in coil 'B' due to change of current in coil 'A' by changing the position of the rheostat.



Mutual Inductance :→

It is defined as the emf induced in coil 'B' due to change of current in coil 'A' is the ratio of flux linkage in coil 'B' to 1 amp. Of current in coil 'A'.

Co-efficient of Mutual Inductance (M)

Coefficient of mutual inductance between the two coils is defined as the weber-turns in one coil due to one ampere current in the other.

1st Method for 'M' :→

$$M = \frac{N_2 \varphi_1}{I_1}$$

N_2 = Number of turns

M = Mutual Inductance

φ_1 = flux linkage

I_1 = Current in ampere

2nd Method for M :→

We know that

$$M = \frac{N_2 \varphi_1}{I_1}$$

$$\Rightarrow MI_1 = N_2\varphi_1$$

$$\Rightarrow -MI_1 = N_2\varphi_1$$

$$\Rightarrow -M\frac{dI_1}{dt} = -N_2\frac{d\varphi_1}{dt}$$

$$\Rightarrow -M\frac{dI_1}{dt} = e_M$$

$$\Rightarrow M\frac{dI_1}{dt} = -e_M$$

$$\Rightarrow M = \frac{-e_M}{\frac{dI_1}{dt}}$$

Where $e_M = -N_2\frac{d}{dt}\varphi_1$ is known as mutually induced emf.

$$e_M = -1volt$$

Then M = 1 Henry

A coil is said to be a mutual inductance of 1 Henry when 1 volt is induced when the current of 1 amp/sec. is changed in its neighbouring coil.

3rd Method for M :→

$$M = \frac{M_o M_r A N_1 N_2}{l}$$

Co-efficient of Coupling :

Consider two magnetically coupled coils having N_1 and N_2 turns respectively. Their individual co-efficient of self-inductances are

$$L_1 = \frac{M_o M_r A N_2^2}{l}$$

$$L_2 = \frac{M_o M_r A N_2^2}{l}$$

The flux φ_1 produced in coil 'A' due to a current of I_1 ampere is

$$\varphi_1 = \frac{L_1 I_1}{N_1} = \frac{M_o M_r A N_1^2}{l} \times \frac{I_1}{N_1}$$

$$\varphi_1 = \frac{M_o M_r A N_1 I_1}{l}$$

Suppose a fraction of this flux i.e. $K_1\varphi_1$ is linked with coil 'B'

Then $M = \frac{K_1\varphi_1}{I_1} \times N_2 = \frac{K_1 N_1 N_2}{l / M_o M_r A}$ ----------(1)

Similarly the flux φ_2 produced in coil 'B' due to I_2 amp. Is

$$\varphi_2 = \frac{M_1 M_r A N_2 I_2}{l}$$

Suppose a fraction of this flux i.e. $K_2\varphi_2$ is linked with coil 'A'

Then $M = \frac{K_2\varphi_2}{I_2} \times N_1 = \frac{K_2 N_2 N_1}{l / M_o M_r A}$ ----------(2)

Multiplying equation (1) & (2)

$$M^2 = \frac{K_1 K_2 N_1^2 N_2^2}{l^2 / M_0^2 M_r^2 A^2} \times N_1$$

$$= K^2 \frac{M_0 M_r A N_1}{l} \quad \frac{M_0 M_r A N_2}{l}$$

$$[\because K_1 = K_2 = K]$$

$$M^2 = K^2 . L_1 . L_2$$

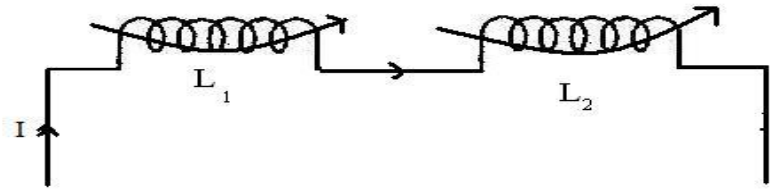
$$K^2 = \frac{M^2 .}{L_1 . L_2}$$

$$\Rightarrow K = \sqrt{\frac{M.}{L_1 . L_2}}$$

Where 'K' is known as the co-efficient of coupling.

Co-efficient of coupling is defined as the ratio of mutual inductance between two coils to the square root of their self- inductances.

Inductances In Series (Additive) :→



Fluxes are in the same durection

Let M = Co-efficient of mutual inductance

L_1 = Co-efficient of self-inductance of first coil.

L_2 = Co-efficient of self-inductance of second coil.

EMF induced in first coil due to self-inductance

$$e_{L_1} = -L_1 \frac{dI}{dt}$$

Mutually induced emf in first coil

$$e_{M_1} = -M \frac{dI}{dt}$$

EMF induced in second coil due to self induction

$$e_{L_2} = -L_2 \frac{dI}{dt}$$

Mutually induced emf in second coil

$$e_{M_2} = -M \frac{dI}{dt}$$

Total induced emf

$$e = e_{L_1} + e_{L_2} + e_{M_1} + e_{M_2}$$

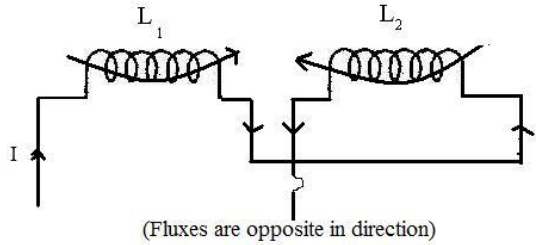
If 'L' is the equivalent inductance, then

$$-L\frac{dI}{dt} = -L_1\frac{dI}{dt} - M\frac{dI}{dt} - L_2\frac{dI}{dt} - M\frac{dI}{dt}$$

$$\Rightarrow -L\frac{dI}{dt} = -\frac{dI}{dt}(L_1 - L_2 - 2M)$$

$$\Rightarrow L = L_1 + L_2 + 2M$$

Inductances In Series (Substnactive) :→



Let M = Co-efficient of mutual inductance

L_1 = Co-efficient of self-inductance of first coil

L_2 -= Co-efficient of self-inductance of second coil Emf induced in first coil due to self induction,

$$e_{L1} = -L_1\frac{dI}{dt}$$

Mutually induced emf in first coil

$$e_{M1} = --M\frac{dI}{dt} = M\frac{dI}{dt}$$

Emf induced in second coil due to self-induction

$$e_{L2} = -L_2\frac{dI}{dt}$$

Mutually induced emf in second coil

$$e_{M2} = --M\frac{dI}{dt} = M\frac{dI}{dt}$$

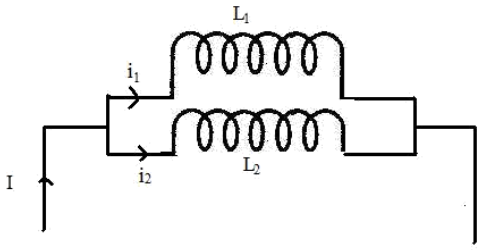
Total induced emf

$$e = e_{L1} + e_{L2} + e_{M1} + e_{M2}$$

Then $-L\frac{dI}{dt} = -L_1\frac{dI}{dt} - L_2\frac{dI}{dt} + M\frac{dI}{dt} + M\frac{dI}{dt}$

$$\Rightarrow -L\frac{dI}{dt} = -\frac{dI}{dt}(L_1 + L_2 - 2M) \qquad \Rightarrow L = L_1 + L_2 - 2M$$

Inductances In Parallel :→



Let two inductances of L_1 & L_2 are connected in parallel

Let the co-efficent of mutual inductance between them is M.

$$I = i_1 + i_2$$

$$\frac{dI}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \qquad (1)$$

$$e = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$= L_2 \frac{di}{dt^2} + M \frac{di}{dt^1}$$

$$\Rightarrow L_1 \frac{di}{dt^1} + M \frac{di}{dt^2} = L_2 \frac{di}{dt^2} + M$$

$$\frac{di}{dt^1} \Rightarrow (L_1 - M) \frac{di}{dt^1} = (L_2 - M) \frac{di}{dt^2}$$

$$\Rightarrow \frac{di_1}{dt} = \frac{(L_2 - M)}{(L_1 - M)} \frac{di_2}{dt} \qquad (2)$$

$$\frac{dI}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$= \frac{(L_2 - M)}{(L_1 - M)} \frac{di_2}{dt} + \frac{di_2}{dt}$$

$$\Rightarrow \frac{dI}{dt} = \frac{L_2 - M}{L_1 - M} + 1 \frac{di}{dt} \qquad \text{----------(3)}$$

If 'L' is the equivalent inductance

$$e = L \frac{di}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$L \frac{di}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$\Rightarrow \frac{di}{dt} = \frac{1}{L} L_1 \frac{di}{dt} + M \frac{di}{dt} \qquad \text{----------(4)}$$

Substituting the value of $\frac{di_1}{dt}$

$$\frac{di}{dt} = \frac{1}{L} L_1 \frac{L_2 - M}{L_1 - M} + M \frac{di_2}{dt} \qquad \text{----------(5)}$$

Equating equation (3) & (5)

$$\frac{L_2-M}{L_1-M}+1\frac{di_2}{dt}=\frac{1}{L}L_1\ \frac{L_2-M}{L_1-M}+M\frac{di_2}{dt}$$

$$\Rightarrow \frac{L_2-M}{L_1-M}+1=\frac{1}{L}L_1\ \frac{L_2-M}{L_1-M}+M$$

$$\Rightarrow \frac{L_2-M+L_1-M}{L_1-M}=\frac{1}{L}\frac{L_1L_2}{L_1-M}\frac{-L_1M+L_1M-M^2}{L_1-M}$$

$$\Rightarrow \frac{L_1+L_2-2M}{L_1-ML_1-M}=\frac{1}{L}\frac{L_1L_2-M^2}{}$$

$$\Rightarrow L_1+L_2-2M=\frac{1}{L}[L_1L_2-M^2]$$

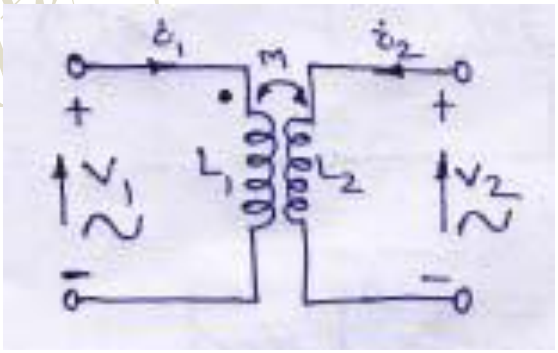
$$\Rightarrow L=\frac{LL-M^2}{L+L-2M}$$

When mutual field assist.

$$L=\frac{LL-M^2}{L+L+2M}$$

When mutual field opposes.

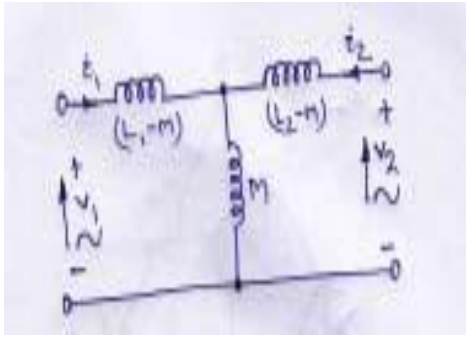
CONDUCTIVELY COUPLED EQUIVALENT CIRCUITS



⇒ The Loop equation are from fig(a)

$$V_1=L_1\frac{di}{dt}+M\frac{di}{dt}_2$$

$$V_2=L_2\frac{di}{dt}_2+M\frac{di}{dt}$$



⇒ The loop equation are from fig(b)

$$V_1=(L_1-M)\frac{di}{dt}^1\mp M\frac{d}{dt}(i_1+i_2)$$

$$V_2 = (L_2 - M)\frac{di}{dt^2} + M\frac{d}{dt}(i_1 + i_2)$$

Which, on simplification become

$$V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} +$$

$$V_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

So called conductively equivalent of the magnetic circuit . Here we may represent $Z_A = L_1\text{-}M$.

$Z_B = (L_2\text{-}M)$ and $Z_C = M$

In case M is + ve and both the currents then $Z_A = L_1\text{-}M$, $Z_B = L_2\text{-}M$ and $Z_C = M$, also , if is – ve and currents in the common branch opposite to each other $Z_A = L_1+M$, $Z_B = L_2+M$ and $Z_C = -M$.

Similarly, if M is –ve but the two currents in the common branch are additive, then also.

$Z_A = L_1+M$, $Z_B = L_2+M$ and $Z_C = -M$.

Further Z_A , Z_B and Z_C may also be assumed to be the T equivalent of the circuit.

Exp. -01 :

Two coupled cols have self inductances $L_1 = 10\times10^{-3}$H and $L_2 = 20\times10^{-3}$H. The coefficient of coupling (K) being 0.75 in the air, find voltage in the second coil and the flux of first coil provided the second coils has 500 turns and the circuit current is given by $i_1 = 2\sin 314.1$A.

Solution :

$$M = K\sqrt{L_1L_2}$$

$$M = 0.75\sqrt{10\times10^{-3}\times 20\times10^{-3}} \Rightarrow M = 10.6\times10^{-3}\ H$$

The voltage induced in second coil is

$$v_2 = M\frac{di_1}{dt} = M\frac{di}{dt}$$

$$= 10.6\times10^{-3}\frac{d}{dt}(2\sin 314t)$$

$$= 10.6\times10^{-3}\times 2\times 314\cos 314t.$$

The magnetic CKt being linear,

$$M = \frac{N\varphi_2}{i_1} = \frac{500\times(K\varphi_1)}{i_1}$$

$$\varphi = \frac{M}{500\times K}\times i_1 = \frac{10.6\times10^{-3}}{500\times 0.75}\times 2\sin 314t$$

$$= 5.66\times10^{-5}\sin 314t$$

$\varphi_1 = 5.66 \times 10^{-5} \sin s\, 314t.$

Exp. 02

Find the total inductance of the three series connected coupled coils.Where the self and mutual inductances are L_1=1H,L_2=2H,L_3=5H

M_{12}= 0.5H, M_{23} = 1H, M_{13} = 1H

Solution:

L_A = L_1+M_{12}+M_{13}
= 1 + 20.5 +1
= 2.5H

= 2+1+0.5
= 3.5H

= 5+1+1
= 7H

= 2.5+3.5+7
= 13H (Ans)

Example 03:

Two identical 750 turn coils A and B lie in parallel planes. A current changing at the rate of 1500A/s in A induces an emf of 11.25 V in B. Calculate the mutual inductance of the arrangement .If the self inductance of each coil is 15mH, calculate the flux produced in coil A per ampere and the percentage of this flux which links the turns of B.

Solution: We know that

$$e_M = \frac{M dI_1}{dt}$$

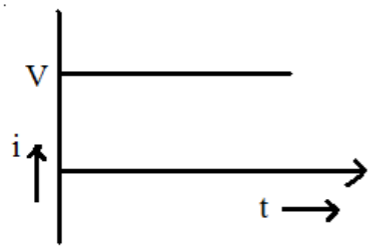
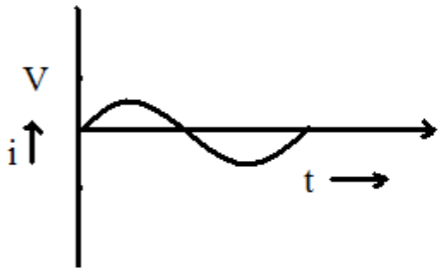
$$M = e_M \Big/ dI_1/dt = \frac{11.25}{1500} = 7.5mH$$

now,

$$L_1 = \frac{N_1\varphi_1}{I_1} = \frac{\varphi_1}{I_1} = \frac{L_1}{N_1} = 15 * \frac{10^{-3}}{750} = 2 * 10^{-5}\text{Wb/A}$$

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{7.5 * 10^{-3}}{15 * 10^{-3}} = 0.5 = 50\%$$

A.C FUNDAMENTAL

Direct Current	Alternating Current
	
(1) D.C. always flow in one direction and whose magnitude remains constant.	(1) A.C. is one which reverse periodically in direction and whose magnitude undergoes a definite cycle changes in definite intervals of time.
(2) High cost of production.	(2) Low cost of production
(3) It is not possible by D.C. Because D.C. is dangerous to the transformer.	(3) By using transformer A.C. voltage can be decreased or increased.
(4) Its transmission cost is too high.	(4) A.C. can be transmitted to a long distance economically.

Definition of A.C. terms :-

Cycle : It is one complete set of +ve and –ve values of alternating quality spread over 360° or 2∏ radan.

Time Period : It is defined as the time required to complete one cycle.

Frequency : It is defined as the reciprocal of time period. i.e. f = 1/ T

Or

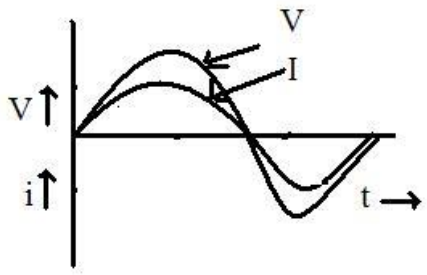
It is defined as the number of cycles completed per second.

Amplitude : It is defined as the maximum value of either +ve half cycle or –ve half cycle.

Phase : It is defined as the angular displacement between two haves is zero.

OR

Two alternating quantity are in phase when each pass through their zero value at the same instant and also attain their maximum value at the same instant in a given cycle.



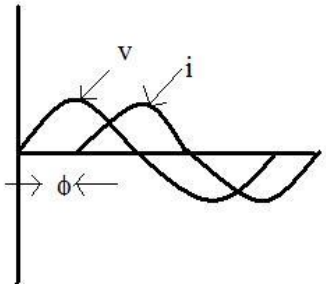
$V = V_m \sin wt$

$i = I_m \sin wt$

Phase Difference :- It is defined as the angular displacement between two alternating quantities.

OR

If the angular displacement between two waves are not zero, then that is known as phase difference. i.e. at a particular time they attain unequal distance.



OR

Two quantities are out of phase if they reach their maximum value or minimum value at different times but always have an equal phase angle between them.

Here $V = V_m \sin wt$

$i = I_m \sin (wt-\varphi)$

In this case current lags voltage by an angle 'φ'.

Phasor Diagram :

Generation of Alternating emf :-

Consider a rectangular coil of 'N" turns, area of cross-section is 'A' nt^2 is placed in x-axis in an uniform magnetic field of maximum flux density *Bm web/nt*^{2}. The coil is rotating in the magnetic field with a velocity of w radian / second. At time t = 0, the coil is in x-axis. After interval of time 'dt' second the coil make rotating in anti-clockwise direction and makes an angle 'θ' with x-direction. The perpendicular component of the magnetic field is $\varphi = \varphi n \cos wt$

According to Faraday's Laws of electro-magnetic Induction

$$e = -N\frac{d}{dt}\varphi$$

$$= -N\frac{d}{dt}(\varphi_m \cos wt)$$

$$= -N(-\varphi_m w \cos wt)$$

$$= Nw\varphi_m \sin wt$$

$$= 2\pi fN\varphi_m \sin wt (\because w = 2\pi f)$$

$$= 2\pi fNB_m A \sin wt$$

$$e = E_m \sin wt$$

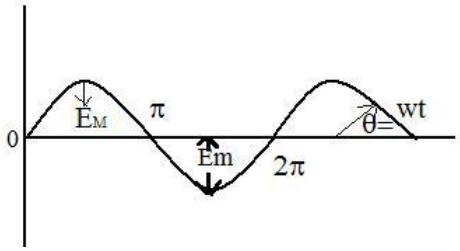
Where $E_m = 2\pi fNB_m A$

f →frequency in Hz

B_m→ Maximum flux density in Wb/mt^2 Now when θ or wt = 90°

$e = E_m$

i.e. $E_m = 2\pi fNB_mA$



Root Mean Square (R.M.S) Value :→

The r.m.s. value of an a.c. is defined by that steady (d.c.) current which when flowing through a given circuit for a given time produces same heat as produced by the alternating current when flowing through the same circuit for the same time.

Sinuscdial alternating current is

$i = I_m \sin wt = I_m \sin \theta$

The mean of squares of the instantaneous values of current over one complete cycle

$$= \frac{2\int_0^{\pi} i^2.d\theta}{(2\pi - 0)}$$

The square root of this value is

$$= \sqrt{\frac{\int_0^{2\pi} i^2.d\theta}{2\pi}}$$

$$= \sqrt{\frac{2\int_0^{\pi} (I_m \sin\theta)^2 d\theta}{2\pi}}$$

$$= \frac{I_m^2}{2\pi}\int_0^{2\pi} \sin^2\theta.d\theta$$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{2\pi}\frac{1-\cos 2\theta}{2}d\theta}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\int_0^{2\pi}(1-\cos 2\theta)d\theta}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\frac{\theta - \sin 2\theta}{2}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\int_0^{2\pi}\left(2\pi - \frac{\sin 4\pi}{2}\right)d\theta}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\int_0^{2\pi}(2\pi - 0)}$$

$$= \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

$$I_{r.m.s} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

Average Value :→

The average value of an alternating current is expressed by that steady current (d.c.) which transfers across any circuit the same charge as it transferred by that alternating current during the sae time.

The equation of the alternating current is $i = I_m \sin\theta$

$$I_{av} = \frac{\int_0^{\pi} i\,.d\theta}{(\pi - 0)}$$

$$= \frac{\int_0^{\pi} I_m.\sin\theta\, d\theta}{\pi} = \frac{I_m}{\pi}\int_0^{\pi}\sin\theta.\,d\theta$$

$$= \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{I_m}{\pi}\left[-\cos\pi - (\cos 0^0)\right]$$

$$= \frac{I_m}{\pi}\left[1 - 0(-1)\right]$$

$$I_{av} = \frac{2}{\pi} I_m$$

$$I_{av} = \frac{2\times \text{Maximum Current}}{\pi}$$

Hence, $I_{av} = 0.637 I_m$

The average value over a complete cycle is zero

Amplitude factor/ Peak factor/ Crest factor :- It is defined as the ratio of maximum value to r.m.s value.

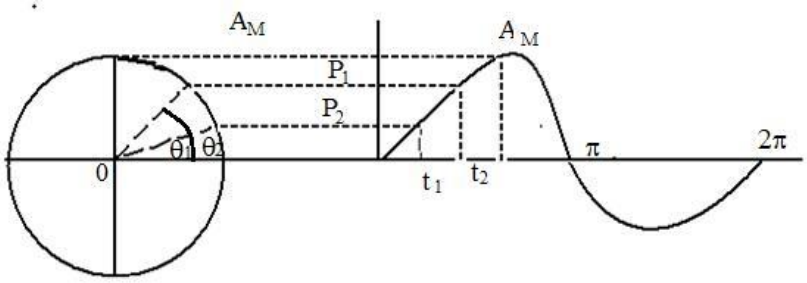
$$Ka = \frac{MaximumValue}{R.M.S.Value} = \frac{I_m}{\frac{I_m}{\sqrt{2}}} = \sqrt{2} = 1.414$$

Form factor : - It is defined as the ratio of r.m.s value to average value.

$$Kf = \frac{r.m.s.Value}{Average.Value} = \frac{0.707 I_m}{0.637 I_m} = \sqrt{2} = 1.414$$

$$Kf = 1.11$$

Phasor or Vector Representation of Alternating Quantity :→



An alternating current or voltage, (quantity) in a vector quantity which has magnitude as well as direction. Let the alternating value of current be represented by the equation $e = E_m \sin wt$. The projection of E_m on Y-axis at any instant gives the instantaneous value of alternating current. Since the instantaneous values are continuously changing, so they are represented by a rotating vector or phasor. A phasor is a vector rotating at a constant angular velocity

At t_1, $e_1 = E_{m1} \sin wt_1$

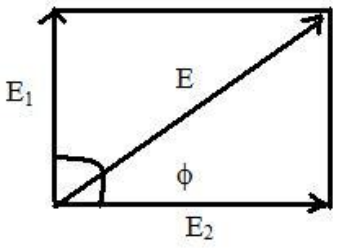
At t_2 , $e_2 = E_{m2} \sin wt_2$

Addition of two alternating Current :→

Let $e_1 = E_{m1} \sin wt$

$e_2 = E_{m2} \sin(wt - \varphi)$

The sum of two sine waves of the same frequency is another sine wave of same frequency but of a different maximum value and Phase.



$$e = \sqrt{e_1^2 + e_2^2 + 2e_1e_2 \cos\varphi}$$

Phasor Algebra :→

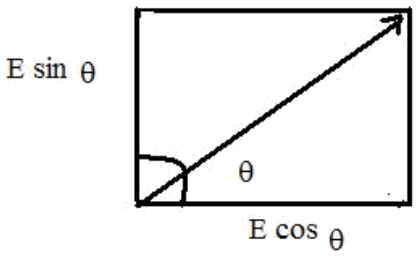
A vector quantity can be expressed in terms of

(i) Rectangular or Cartesian form

(ii) Trigonometric form

(iii) Exponential form

(iv) Polar form



$E = a + jb$

$= E(\cos\theta + j \sin\theta)$

Where a = E cos θ is the active part

b = E sin θ is the reactive part

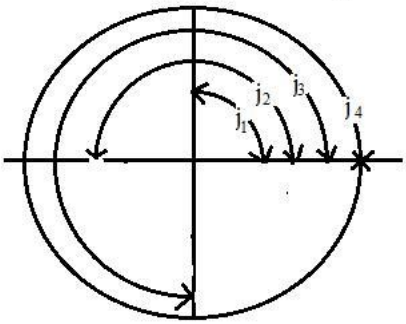
$\theta = \tan^{-1}\frac{b}{a}$ = Phase angle

$j = \sqrt{-1}(90^o)$

$j^2 = -1(180^o)$

$j^3 = -j(270^o)$

$j^4 = 1\ (360^o)$



(i) Rectangular for :-

$E = a \pm jb$

$\tan\theta = b / a$

(ii) Trigonometric form :-

$E = E(\cos\theta \pm j \sin\theta)$

(iii) Exponential form :-

$E = Ee^{\pm j\theta}$

(iv) Polar form :-

$E = E/\pm e(E = \sqrt{a^2 + b^2})$

Addition or Subtration :-

$E_1 = a_1 + jb_1$

$E_2 = a_2 + jb_2$

$E_1 \pm E_2 = (a_1 + a_2) \pm (b_1 + b_2$

$\tan^{-1}\frac{b_1 + b_2}{\varphi} =$

Multiplication : -

$E_1 \times E_2 = (a_1 + ja_1) \pm (a_1 + jb_2)$

$= (a_1a_2 - b_1b_2) + j(a_1a_2 + b_1b_2)$

$$\varphi = \tan^{-1} \frac{a_1 b_2 + b_1 a_2}{a_1 a_2 - b_1 b_2}$$

$E_1 = E_1 \angle \theta_1$

$E_2 = E_2 \angle \theta_2$

$E_1 \times E_2 = E_1 E_2 \quad \angle \varphi_1 + \varphi_2$

Division :-

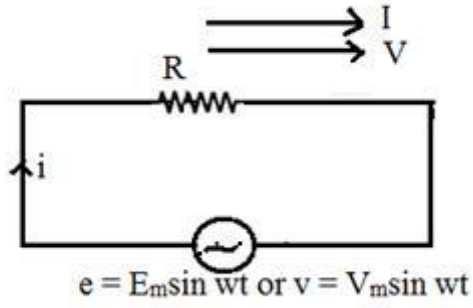
$E_1 = E_1 \angle \theta_1$

$E_2 = E_2 \angle \theta_2$

$$\frac{E_1}{E_2} = \frac{E_1 \angle \theta_1}{E_2 \angle \theta_2} = \frac{E_1}{E_2} \angle \theta_1 - \theta_2$$

A.C. through Pure Resistance :→

Let the resistance of R ohm is connected across to A.C supply of applied voltage



$e = E_m \sin wt$ ----------(1)

Let 'I' is the instantaneous current .

Here $e = iR$

$\Rightarrow i = e/R$

$i = E_m \sin wt / R$----------(2)

By comparing equation (1) and equation (2) we get alternating voltage and current in a pure resistive circuit are in phase

Instantaneous power is given by

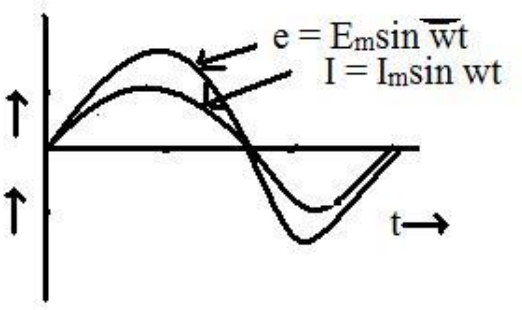
$P = ei$

$= E_m \sin wt \cdot I_m \sin wt = E_m I_m \sin^2 wt$

$= \frac{E_m I_m}{2} .2\sin^2 wt$

$= \frac{E_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} .(1 - \cos 2wt)$

$P = \frac{E_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} - \frac{E_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} .\cos 2wt$



i.e. $P = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} - \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} .\cos 2wt$

Where $\frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$ is called constant part of power.

$\frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos 2wt$ is called fluctuating part of power.

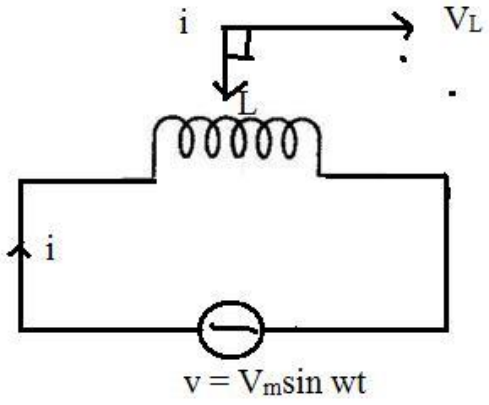
The fluctuating part $\frac{V_m I_m}{2} \cos 2wt$ of frequency double that of voltage and current 2 waves.

Hence power for the whole cycle is $P = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = V_{rms} \cdot I_{rms}$

$\Rightarrow P = VI$ watts

A.C through Pure Inductance :→

Let inductance of 'L' henry is connected across the A.C. supply



$v = V_m \sin wt$ ---------- (1)

According to Faraday's laws of electromagnetic inductance the emf induced across the inductance

$$V = L \frac{di}{dt}$$

$\frac{di}{dt}$ is the rate of change of current

$$V_m \sin wt = L \frac{di}{dt}$$

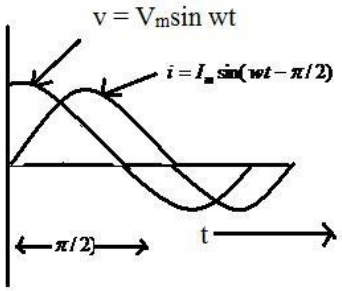
$$dt = \frac{L}{V_m} \sin wt.dt$$

$$\Rightarrow di = \frac{V_m}{L} \sin wt.dt$$

Integrating both sides,

$$\int di = \int \frac{V_m}{L} \sin wt.dt$$

$$i = \frac{V_m}{L} - \frac{\cos wt}{w}$$



$$i = -\frac{V_m \cos wt}{wL}$$

$$i = -\frac{V_m}{wL}\cos wt$$

$$i = -\frac{V_m}{wL}\sin\left(wt - \frac{\pi}{2}\right)$$

$$= -\frac{V_m}{X_L}\sin\left(wt - \frac{\pi}{2}\right) \quad [\because X_L = 2\pi fL = wL]$$

Maximum value of i is

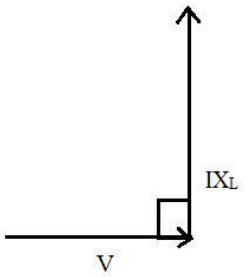
$I_m = \frac{V_m}{X_L}$ when $\sin\left(wt - \frac{\pi}{2}\right)$ is unity.

Hence the equation of current becomes $i = I_m \sin(wt - \pi/2)$

So we find that if applied voltage is rep[resented by $v = V_m \sin wt$, then current flowing in a purely inductive circuit is given by

$$i = I_m \sin(wt - \pi/2)$$

Here current lags voltage by an angle π/2 Radian.

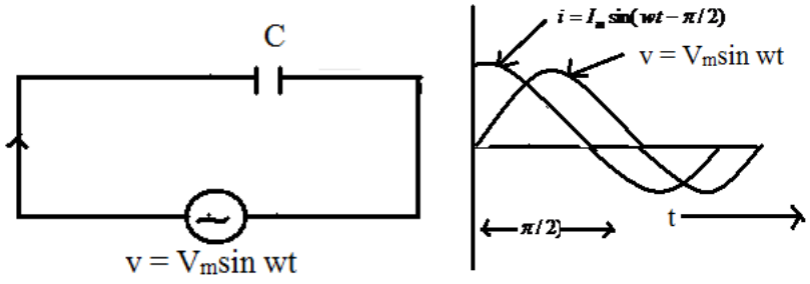


Power factor = cos φ
= cos 90°
= 0

Power Consumed = VI cos φ
= VI×0
= 0

Hence, the power consumed by a purely Inductive circuit is zero.

A.C. Through Pure Capacitance : →



Let a capacitance of 'C" farad is connected across the A.C. supply of applied voltage

$$v = V_m \sin wt \text{ ---------- (1)}$$

Let 'q' = change on plates when p.d. between two plates of capacitor is 'v'

$$q = cv$$

$$q = cV_m \sin wt$$

$$\frac{dq}{dt} = c\frac{d}{dt}(V_m \sin wt)$$

$$i = cV_m \sin wt$$

$$= wcV_m \cos wt$$

$$= \frac{V_m}{1/wc} = \cos wt$$

$$= \frac{V_m}{Xc} = \cos wt$$

[Q $X_c = \frac{1}{wc} = \frac{1}{2\pi fc}$ is known as capacitive reactance in ohm.]

$$= I_m \cos wt$$

$$= I_m \sin(wt + \pi/2)$$

Here current leads the supply voltage by an angle π/2 radian.

Power factor = cos φ

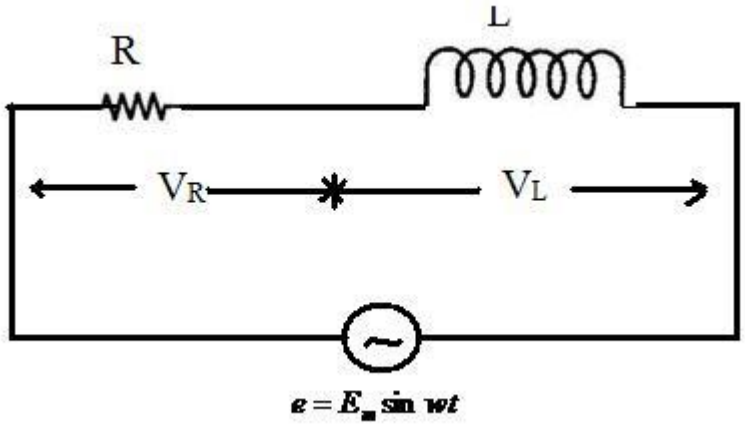
= cos 90° = 0

Power Consumed = VI cos φ

=VI×0 =0

The power consumed by a pure capacitive circuit is zero.

A.C. Through R-L Series Circuit : →



The resistance of R-ohm and inductance of L-henry are connected in series across the A.C. supply of applied voltage

$$e = E_m \sin wt \quad \text{.....} \quad (1)$$

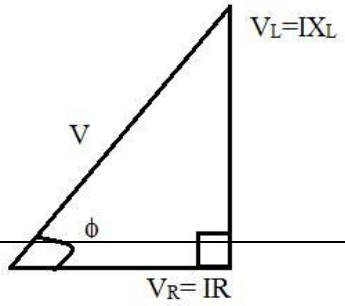
$$V = V_R + jV_L$$

$$= \sqrt{V_R^2 + V_L^2} \quad \angle\varphi = \tan^{-1}\frac{X_L}{R}$$

$$= \sqrt{(IR)^2 + (IX_L)^2} \quad \angle\varphi = \tan^{-1}\frac{X_L}{R}$$

$$= I\sqrt{R^2 + X_L^2} \quad \angle\varphi = \tan^{-1}\frac{X_L}{R}$$

$$V = IZ\angle\varphi = \tan^{-1}\frac{X_L}{R}$$



Where $Z = \sqrt{R^2 + X_L{}_2}$

$= R + jX_L$ is known as impedance of R-L series Circuit.

$$I = \frac{V}{Z\angle\varphi} = \frac{E_m \sin wt}{Z\angle\varphi}$$

$$I = I_m \sin(wt - \varphi)$$

Here current lags the supply voltage by an angle φ.

Power Factor :→ It is the cosine of the angle between the voltage and current.

OR

It is the ratio of active power to apparent power.

OR

It is the ratio of resistance to inpedence .

Power :→

$= v.i$

$= V_m \sin wt . I_m \sin(wt - \varphi)$

$= V_m I_m \sin wt . \sin(wt - \varphi)$

$= \frac{1}{2} V_m I_m 2\sin wt . \sin(wt - \varphi)$

$= \frac{1}{2} V_m I_m [\cos\varphi - \cos 2(wt - \varphi)]$

Obviously the power consists of two parts.

(i) a constant part $\frac{1}{2} V_m I_m \cos\varphi$ which contributes to real power.

(ii) a pulsating component $\frac{1}{2} V_m I_m \cos(2wt - \varphi)$ which has a frequency twice that of the voltage and current. It does not contribute to actual power since its average value over a complete cycle is zero.

Hence average power consumed

$$= \frac{1}{2} V_m I_m \cos\varphi$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi$$

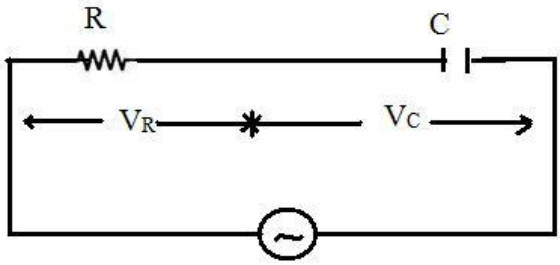
$$= VI \cos\varphi$$

Where V & I represents the r.m.s value.

A.C. Through R-C Series Circuit : →

The resistance of 'R'-ohm and capacitance of 'C' farad is connected across the A.C. supply of applied voltage

$e = E_m \sin wt$---------- (1)



$V = V_R + (-jV_C)$

$= IR + (-jIX_C)$

$= I(R - jX_C)$

$V = IZ$

Where $Z = R - jX_C = \sqrt{R^2 + X_C{}^2}$ is known as impedance of R-C series Circuit.

$Z = R - jX_C$

$= \sqrt{R^2 + X_C^2}$

$\angle -\varphi = \tan^{-1} \frac{X_C}{R}$

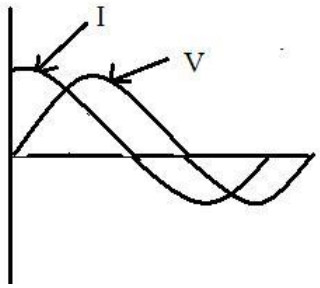
$V = IZ\angle -\varphi$

$\Rightarrow I = \frac{V}{Z\angle -\varphi}$

$= \frac{E_m \sin wt}{Z\angle -\varphi}$

$= \frac{E}{Z}\angle^m \sin(wt + \varphi)$

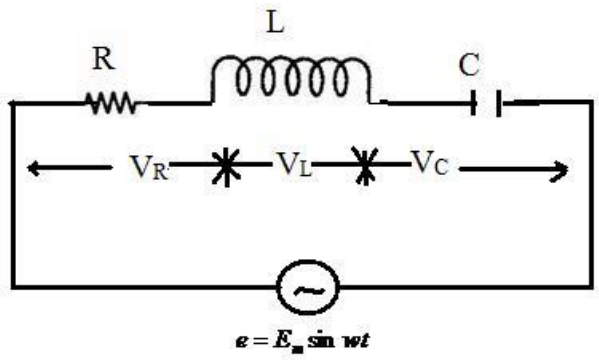
$\Rightarrow I = I_m \sin(wt + \varphi)$



Here current leads the supply voltage by an angle 'φ'.

A.C. Through R-L-C Series Circuit : →

Let a resistance of 'R'-ohm inductance of 'L' henry and a capacitance of 'C' farad are connected across the A.C. supply in series of applied voltage



$e = E_m \sin wt$ ---------- (1)

$$\vec{e} = \vec{V_R} + \vec{V_L} + \vec{V_C}$$
$$= V_R + jV_L - jV_C$$
$$= V_R + j(V_L - V_C)$$
$$= I_R + j(IX_L - IX_C)$$
$$= I[R + j(X_L - X_C)]$$
$$= I\sqrt{R^2 + (X_L - X_C)^2} \quad \angle \pm \varphi = \tan^{-1}\frac{X_L - X_C}{R}$$
$$= IZ\angle \pm \varphi$$

Where $Z = I\sqrt{R^2 + (X_L - X_C)^2}$ is known as the impedance of R-L-C Series Circuit.

If

If $X_L > X_C$, then the angle is +ve.
$X_L < X_C$, then the angle is -ve.

Impedance is defined as the phasor sum of resistance and net reactance

$$e = IZ\angle \pm \varphi$$
$$\Rightarrow I = \frac{e}{Z\angle \pm \varphi} IZ\angle \pm \varphi \quad = \frac{E_m \sin wt}{Z\angle \pm \varphi} = I_m \sin(wt \pm \varphi)$$

(1) If $X_L > X_C$, then P.f will be lagging.

(2) If $X_L < X_C$, then, P.f will be leading.

(3) If $X_L = X_C$, then, the circuit will be resistive one. The p.f. becomes unity and the resonance occurs.

REASONANCE

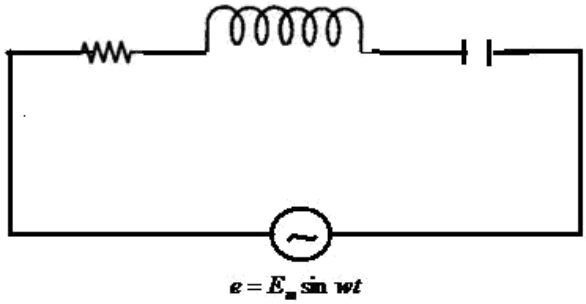
It is defined as the resonance in electrical circuit having passive or active elements represents a particular state when the current and the voltage in the circuit is maximum and minimum with respect to the magnitude of excitation at a particular frequency and the impedances being either minimum or maximum at unity power factor

Resonance are classified into two types.

(1) Series Resonance

(2) Parallel Resonance

(1) Series Resonance :- Let a resistance of 'R' ohm, inductance of 'L' henry and capacitance of 'C' farad are connected in series across A.C. supply



$e = E_m \sin wt$

The impedance of the circuit

$$Z = R + j(X_L - X_C)]$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The condition of series resonance:

The resonance will occur when the reactive part of the line current is zero The p.f. becomes unity.

The net reactance will be zero.

The current becomes maximum.

At resonance net reactance is zero

$$X_L - X_C = 0$$

$$\Rightarrow X_L = X_C$$

$$\Rightarrow W_o L = \frac{1}{W_o C}$$

$$\Rightarrow W_o^2 LC = 1$$

$$\Rightarrow W_o^2 = \frac{1}{LC}$$

$$\Rightarrow W_o = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow 2\pi f_o = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow f_o = \frac{1}{2\pi\sqrt{LC}}$$

Resonant frequency $(f_o) = \frac{1}{2\pi\sqrt{LC}}$

Impedance at Resonance

$Z_0 = R$

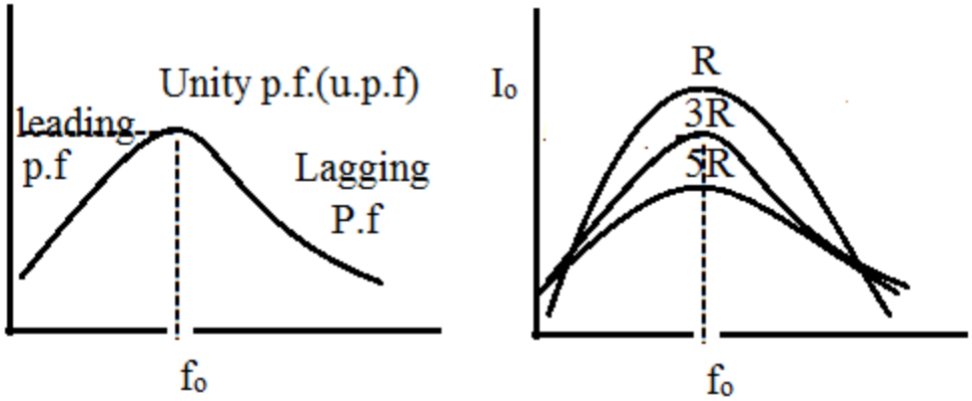
Current at Resonance

$$I_o = \frac{V}{R}$$

Power factor at resonance

$$p.f. = \frac{R}{Z_o} = \frac{R}{R} = 1 \qquad [\text{Q} Z_o = R]$$

Resonance Curve :-



At low frequency the X_c is greater and the circuit behaves leading and at high frequency the X_L becomes high and the circuit behaves lagging circuit.

If the resistance will be low the curve will be stiff (peak).

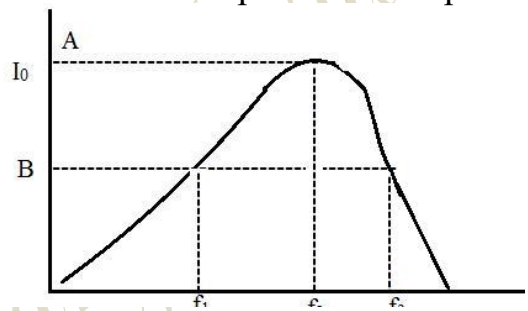
- If the resistance will go oh increasing the current goes on decreasing and the curve become flat.

Band Width :→

At point 'A' the power loss is I_0^2R.

The frequency is f_0 which is at resonance. $\frac{I^2 R}{2}$

The power loss is 50% of the power loss at point 'A"/



Hence the frequencies corresponding to point 'B' is known as half power frequencies f_1 & f_2. f_1 = Lower half power frequency

$$f_1 = f_0 - \frac{R}{4\pi L}$$

F_2 = Upper half power frequency

$$f_2 = f_0 + \frac{R}{4\pi L}$$

Band width (B.W.) is defined as the difference between upper half power frequency ad lower half power frequency.

$$\text{B.W.} = f_2 - f_1 = \frac{R}{2\pi L}$$

Selectivity : →

Selectivity is defined as the ratio of Band width to resonant frequency

Selectivity = $\frac{B.W.}{f_0} = \frac{R}{2\pi L}$ $\quad Selectivity = \frac{R}{2\pi f_0 L}$

Quality Factor (Q-factor) :→

It is defined as the ratio of 2π × Maximum energy stored to energy dissipated per cycle

Q-factor $= \frac{2\pi \times \frac{1}{2} LI_0^2}{I^2 RT}$

$= \frac{\pi L\left(\sqrt{2}I\right)^2}{I^2 RT}$

$= \frac{\pi L.2I^2}{I^2 RT}$

$= \frac{\pi L.2I^2}{I^2 RT}$

$= \frac{2\pi L.}{RT}$

Quality factor = = $\frac{2\pi f_0 L.}{R}$

$Q = \frac{1.}{I_0} = f$

Quality factor is defined as the reciprocal of power factor.

Q factor = = $\frac{1.}{\cos \varphi}$

It is the reciprocal of selectivity.

Q-factor Or Magnification factor $= \frac{\text{Voltage across Inductor.}}{\text{Voltage across resistor}}$

$= \frac{I_0 X_L}{I_0 R}$

$= \frac{X_L}{R}$

$= \frac{2\pi f_0 L}{RR} = \frac{W_0 L}{R}$

Q- factor = = $\frac{W_0 L}{R}$

Q-factor factor $= \frac{\text{Voltage across Capacotor.}}{\text{Voltage across resistor}}$

$= \frac{I_0 X_c}{I_0 R}$

$$= \frac{X_C}{R}$$

$$= \frac{\frac{1}{2\pi f_0 C}}{} = \frac{1}{2\pi f_0 CR}$$

$$\text{Q-factor} = \frac{1}{W_0 CR}$$

$$Q^2 = \frac{W_0 L}{R} \times \frac{1}{W_0 CR}$$

$$Q^2 = \frac{1}{R^2 C}$$

$$Q = \sqrt{\frac{1}{R^2 C}}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

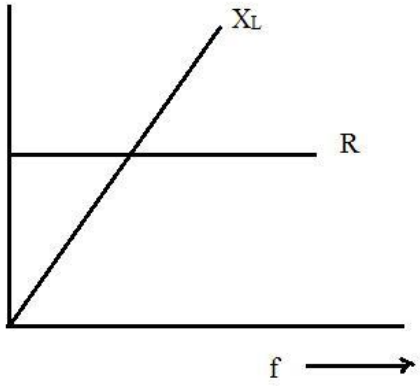
Graphical Method :→

(1) Resistance is independent of frequency It represents a straight line.

(2) Inductive Reactance $X_L = 2\pi fL$

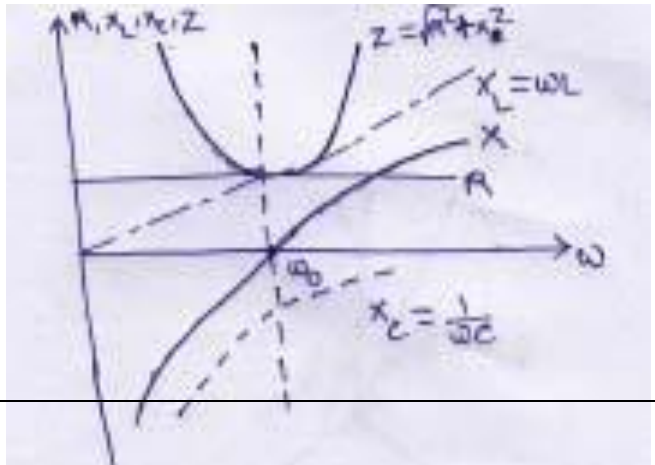
It is directly proportional to frequency. As the frequency increases , X_L increases

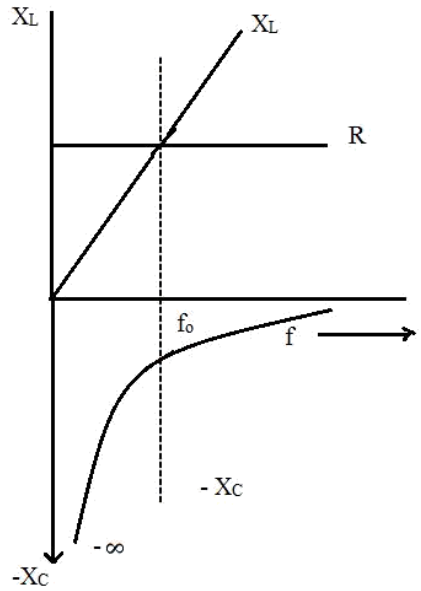
(3) Capacitive Reactance $X_C = \frac{1}{2\pi fC}$



It is inversely proportional to frequency. As the frequency increases, X_C decreases.

When frequency increases, X_L increases and X_C decreases from the higher value.



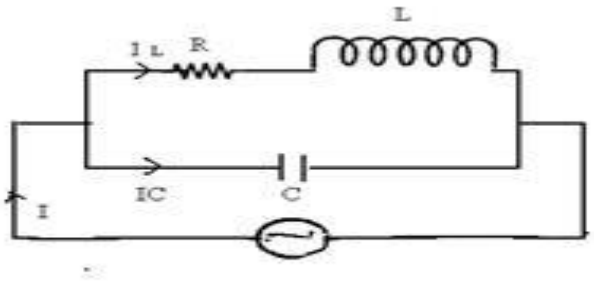


At a certain frequency. $X_L = X_C$

That particular frequency is known as Resonant frequency.

Variation of circuit parameter in series resonance:

(2) Parallel Resonance :- Resonance will occur when the reactive part of the line current is zero.



At resonance,

$I_C - I_L \sin \varphi = 0$

$I_C = I_L \sin\varphi$

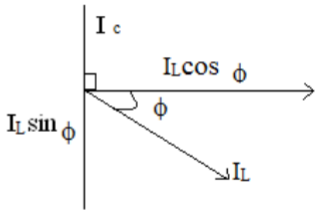
$$\Rightarrow \frac{V}{X_C} = \frac{V}{R^2 + X_L{}^2} \sin \varphi$$

$$\Rightarrow \frac{V}{X_C} = \frac{V}{\sqrt{R^2 + X_L^2}} \times \frac{X_L}{\sqrt{R^2 + X_L^2}}$$

$$\Rightarrow \frac{1}{X_C} = \frac{X_L}{R^2 + X_L{}^2}$$

$$\Rightarrow R^2 + X_L^2 = X_L . X_C$$

$$\Rightarrow Z^2 == X_L . X_C = W_0 L \times W \frac{1}{C_0}$$



$$Z^2 = \frac{L}{C}$$

$$\Rightarrow R^2 + X_L^2 = \frac{L}{C}$$

$$\Rightarrow R^2 + (2\pi f_0 L)^2 = \frac{L}{C} \Rightarrow$$

$$R^2 + 4\pi^2 f_0^2 L^2 = \frac{L}{C}$$

$$\Rightarrow 4\pi^2 f_0^2 L^2 = \frac{L}{C} - R^2$$

$$\Rightarrow f_0^2 = \frac{1}{4\pi^2 f_0^2 L^2} = \frac{L}{C} - R^2$$

$$\Rightarrow f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

f_0 = Resonant frequency in parallel circuit.

Current at Resonance = $I_L \cos\varphi$

$$= \frac{V}{\sqrt{R^2 + X_L^2}} \cdot \frac{R}{\sqrt{R^2 + X_L^2}}$$

$$= \frac{VR}{R^2 + X_L^2}$$

$$= \frac{VR}{Z^2}$$

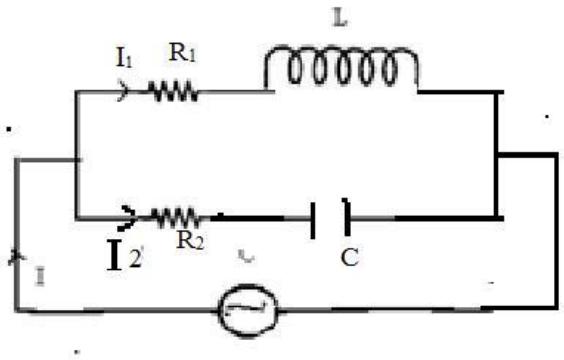
$$= \frac{VR}{L/RC} = \frac{V}{L/RC}$$

$$= \frac{V}{\text{Dynamic Impedence}}$$

$L / RC \rightarrow$ Dynamic Impedance of the circuit.

or, dynamic impedances is defined as the impedance at resonance frequency in parallel circuit.

Parallel Circuit :→



The parallel resonance condition:

When the reactive part of the line current is zero.

The net reactance is zero.

The line current will be minimum.

The power factor will be unity

Impedance $Z_1 = R_1 + jX_L$

$Z_2 = R_2 - jX_C$

Admittance $Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_L}$

$= \frac{(R_1 + jX_L)}{(R_1 + jX_L)(R_1 - jX_L)}$

$= \frac{R_1 + jX_L}{R_1^2 + X_L^2}$

$Y_1 = \frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2}$

Admittance $Y_2 = \frac{1}{Z_2} = \frac{1}{R_1 + jX_C}$

$= \frac{(R_2 + jX_C)}{(R2 - jX_C)(R_2 + jX_C)}$

$= \frac{R_2 + jX_L}{R_2^2 + X_C^2}$

$Y_2 = \frac{R_2}{R_2^2 + X_C^2} + j\frac{X_C}{R_2^2 + X_C^2}$

Total Admittance Admittance $\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$

$\Rightarrow Y = Y_1 + Y_2$

$\Rightarrow Y = \frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2} + \frac{R2}{R_2^2 + X_C^2} + j\frac{X_C}{R_2^2 + X_C^2}$

$\Rightarrow Y = \frac{R_1}{R_1^2 + X_L^2} + \frac{R2}{R_2^2 + X_C^2} - j\left(\frac{X_L}{R_1^2 + X_L^2} - \frac{X_C}{R_2^2 + X_C^2}\right)$

At Resonance,

$\frac{X_L}{R_1^2 + X_L^2} - \frac{X_C}{R_2^2 + X_C^2} = 0$

$\Rightarrow \frac{X_L}{R_1^2 + X_L^2} = \frac{X_C}{R_2^2 + X_C^2}$

$\Rightarrow X_L\left(R_2^2 + X_C^2\right) = X_C\left(R_1^2 + X_L^2\right)$

$\Rightarrow 2\pi fL\,R_2^2 + \frac{1}{4\pi^2 f^2 C} = \frac{1}{2\pi fC}\left(R_1^2 + 4\pi^2 f^2 L^2\right)$

$\Rightarrow 2\pi fLR_2^2 + \frac{L}{2\pi fC^2} = \frac{R^2}{2\pi fC} + \frac{2\pi fL^2}{C}$

$$\Rightarrow \frac{L}{2\pi fC^2} - \frac{R_1^2}{2\pi fC} = \frac{2\pi fL^2}{C} - 2\pi fLR_2^2$$

$$\Rightarrow \frac{1}{2\pi fC}\left(\frac{L}{C} - R_1^2\right) = 2\pi fL\left(\frac{L}{C} - R_2^2\right)$$

$$\Rightarrow 4\pi^2 f^2 LC = \frac{\frac{L}{C} - R_1^2}{\frac{L}{C} - R_2^2} = \frac{L - CR_1^2}{L - CR_2^2}$$

$$\Rightarrow 4\pi^2 f^2 = \frac{1}{LC}\cdot\frac{L - CR_1^2}{L - CR_2^2}$$

$$\Rightarrow f^2 = \frac{1}{4\pi^2 LC}\cdot\frac{L - CR_1^2}{L - CR_2^2}$$

$$\Rightarrow f = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{L - CR_1^2}{L - CR_2^2}}$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{L - CR_1^2}{L^2C - LC^2R_2^2}}$$

f is called Resonant frequency.

If $R^2 = 0$

Then $f = \frac{1}{2\pi}\sqrt{\frac{L - CR_1^2}{LC}}$

$$= \frac{1}{2\pi L}\sqrt{\frac{L - CR_1^2}{C}}$$

$$= \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R_1^2}$$

$$= \frac{1}{2\pi}\sqrt{\frac{L}{L^2C} - \frac{R_1^2}{L^2}}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{L}{LC} - \frac{R_2}{L^2}}}$$

If R_1 and $R_2 = 0$, then

$$f = \frac{1}{2\pi}\sqrt{\frac{L}{L^2C}}$$

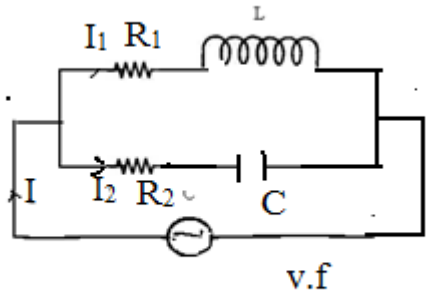
$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi\sqrt{LC}}}$$

Comparison of Series and Parallel Resonant Circuit :→

Item	Series ckt (R-L-C)	Parallel ckt (R– L and C)

™ Impedance at Resonance	Minimum	Maximum
™ Current at Resonance	Maximum= $\frac{V}{R}$	Minimum= $\frac{V}{(L / CR)}$
™ Effective Impedance	R	$\frac{L}{CR}$
™ P.f. at Resonance	Unity	Unity
™ Resonant Frequency	$\frac{1}{2\pi\sqrt{LC}}$	$\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}$
™ It Magnifies	Voltage	Current
™ Magnification factor	$\frac{WL}{R}$	$\frac{WL}{R}$

Parallel circuit :→



$$Z_1 = R_1 + jX_L = \sqrt{R_1^2 + X_L^2}\ \angle\varphi_1$$

$$Z_2 = R_1 - jX_C = \sqrt{R2_1^2 + X_C^2}\ \angle -\varphi_2$$

$$I_1 = \frac{V}{Z_1\angle\varphi_1} = \frac{V}{Z_1}\ \angle -\varphi_1 = I_1\ \angle -\varphi_1$$

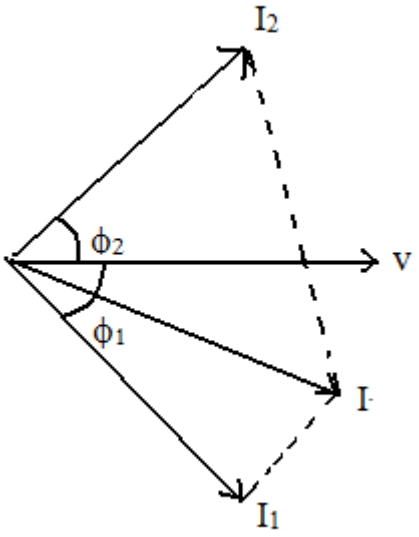
Where $V = VY_1$
Z_1

Here Y_1 → Admittance of the circuit

Admittance is defined as the reciprocal of impedence.

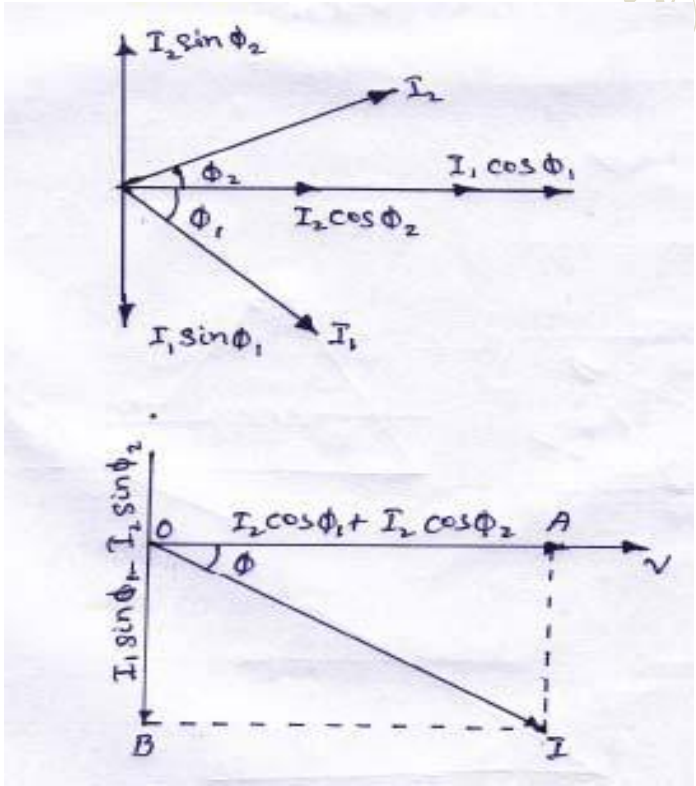
$$I_1 = VY_1 = \frac{v}{R_1 + jX_L}$$

$$I_2 = \frac{V}{Z_2 \angle -\varphi 2_1} = \frac{V}{Z_2} \angle \varphi_1 = VY_2 \angle \varphi = I_2 \angle \varphi_2$$



$$I = \sqrt{I_1^2 + I_2^2} + 2I_1 I_2 \cos(\varphi_1 + \varphi_2)$$

$$I = I_1 \angle -\varphi_1 + I_2 \angle \varphi_2$$



The resultant current "I" is the vector sum of the branch currents I_1 & I_2 can be found by using parallelogram low of vectors or resolving I_2 into their X

– and Y- components (or active and reactive components respectively) and then by combining these components.

Sum of active components of I_1 and $I_2 = I_1 \cos \varphi_1 + I_2 \cos \varphi_2$ Sum of the reactive components of I_1 and $I_2 = I_2 \sin \varphi_2 - I_1 \sin \varphi_1$

EXP–01:

A 60Hz voltage of 230 V effective value is impressed on an inductance of 0.265 H

(i) Write the time equation for the voltage and the resulting current. Let the zero axis of the voltage wave be at t = 0.

(ii) Show the voltage and current on a phasor diagram.

(iii) Find the maximum energy stored in the inductance.

Solution :-

$$V_{max} = \sqrt{2}V = \sqrt{2} \times 230V$$

$$f = 60\text{Hz}, \quad W = 2\pi f = 2\pi \times 60 = 377 rad/s.$$

$$x_l = wl = 377 \times 0.265 = 100\Omega \quad .$$

(i) The time equation for voltage is $V(t) = 230\sqrt{2} \sin 377t.$

$$I_{max} = V_{max} / x_l = 230\sqrt{2} / 100. = 2.3\sqrt{2}$$

$\varphi = 90^o$ (*lag*).

Q Currente quation is.

$$i(t) = 2.3\sqrt{2} \sin(377t - \pi/2)$$

or $= 2.3\sqrt{2} \cos 377t$

(ii) Iti

(iii) or $E_{max} = \frac{1}{2} LI^2_{max} = \frac{1}{2} \times 0.265 \times (2.3\sqrt{2})^2 = 1.4J$

Example -02 :

The potential difference measured across a coil is 4.5 v, when it carries a direct current of 9 A. The same coil when carries an alternating current of 9A at 25 Hz, the potential difference is 24 v. Find the power and the power factor when it is supplied by 50 v, 50 Hz supply.

Solution :

Let R be the d.c. resistance and L be inductance of the coil.

$$R = V/I = 4.5/9 = 0.5\Omega$$

With a.c. current of 25Hz, z = V/1.

$$\frac{24}{9} = 2.66\Omega$$

$$x_l = \sqrt{Z^2 - R^2} = \sqrt{2.66^2 - 0.5^2}$$

$$= 2.62\Omega$$

$$x_l = 2\pi \times 25 \times L$$

$$x_l = 0.0167\Omega$$

At 50Hz

$$x_l = 2.62 \times 2 = 5.24\Omega$$

$$Z = \sqrt{0.5^2 + 5.24^2}$$

$$= 5.06\ \Omega$$

I = 50/5.26 = 9.5 A

$P = I^2/R = 9.5^2 \times 0.5 = 45$ watt.

Example – 03 :

A 50- μf capacitor is connected across a 230-v, 50 – Hz supply. Calculate

(a) The reactance offered by the capacitor.

(b) The maximum current and

(c) The r.m.s value of the current drawn by the capacitor.

Solution :

(a) $$x_l = \frac{1}{wc} = \frac{1}{2\pi fe} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.6\Omega$$

(c) Since 230 v represents the r.m.s value

$$Q\ I_{rms} = 230 / x_l = 230 / 63.6 = 3.62A$$

(b) $$I_m = I_{r.m.s} \times \sqrt{2} = 3.62 \times \sqrt{2} = 5.11A$$

Example – 04 :

In a particular R – L series circuit a voltage of 10v at 50 Hz produces a current of 700 mA. What are the values of R and L in the circuit ?

Solution :

(i) $$Z = \sqrt{R^2 + (2\pi \times 50L)^2}$$

$$= \sqrt{R^2 + 98696L^2}$$

$$V = 1z$$

$$10 = 700 \times 10^{-3}\sqrt{(R^2 + 98696L^2)}$$

$$\sqrt{(R^2 + 98696L^2)} = 10 / 700 \times 10^{-3} = 100 / 7$$

$$R^2 + 98696L^2 = 10000/49 \text{---------- (I)}$$

(ii) In the second case $Z = \sqrt{R_2 + (2\pi \times 75L)_2}$

$$Q10 = 500 \times 10^{-3}\sqrt{R^2 + 222066L^2)} = 20$$

$$\sqrt{R^2 + 222066L^2)} = 20$$

$R_2 + 222066L_2 = 400$ ----------(II)

Subtracting Ea.(I) from (ii), we get,

$222066L^2 - 98696L^2 = 400 - (10000/49)$

$\Rightarrow 123370L^2 = 196$

$\Rightarrow L^2 = \frac{196}{123370}$

$\Rightarrow L = \sqrt{\frac{196}{123370}} = 0.0398H = 40$ mH.

Substituting this value of L in equation (ii) we get $R_2 + 222066L_2 (0.398)_2 = 400$

$\Rightarrow R = 6.9\,\Omega$.

Example – 04 :

A 20Ω resistor is connected in series with an inductor, a capacitor and an ammeter across a 25 –v, variable frequency supply. When the frequency is 400Hz, the current is at its Max^m value of 0.5 A and the potential difference across the capacitor is 150v. Calculate

(a) The capacitance of the capacitor.

(b) The resistance and inductance of the inductor.

Solution :

Since current is maximum, the circuit is in resonance.

$x_l = V_C/1 = 150/0.5 = 300\Omega$

(a) $x_l = 1/2\pi fe \Rightarrow 300 = 1/2\pi \times 400 \times c$

$\Rightarrow c = 1.325 \times 10^{-6} f = 1.325\mu f$.

(b) $x_l = x_l = 150/0.5 = 300\Omega$

$2\pi \times 400 \times L = 300$

$\Rightarrow L = 0.49H$

(c) At resonance,

Circuit resistance = 20+R

$\Rightarrow V/Z = 2510.5$

$\Rightarrow R = 30\Omega$

Exp.-05

An R-L-C series circuits consists of a resistance of 1000Ω, an inductance of 100MH an a capacitance of wμ μf or 10PK

(ii) The half power points.

Solution :

i) $fo = \frac{1}{2\pi\sqrt{10^{-1} \times 10^{-4}}} = \frac{10^6}{2\pi} = 159KHz$

ii) $\varphi = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{1000} \times \sqrt{\frac{10^{-1}}{10^{-11}}} = 100$

iii) $f_1 = fo - \frac{R}{4\pi l} = 159 \times 10^3 - \frac{1000}{4\pi \times 10^{-1}} = 158.2KHz$

$f_2 = fo - \frac{R}{4\pi l} = 159 \times 10^{-3} + \frac{1000}{4\pi \times 10^{-1}} = 159.8KHz.$

Exp. -06

Calculate the impedance of the parallel –turned circuit as shown in fig. 14.52 at a frequency of 500 KHz and for band width of operation equal to 20 KHz. The resistance of the coil is 5Ω.

Solution :

At resonance, circuit impedance is L/CR. We have been given the value of R but that of L and C has to be found from the given the value of R but that of L and C has to be found from the given data.

$BW = \frac{R}{2\pi l}, 20 \times 10^3 = \frac{5}{2\pi \times l} or\ l = 39\mu H$

$fo - \frac{1}{2\pi} = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \frac{1}{2\pi}\sqrt{\frac{1}{39\times10^{-6} C} - \frac{5^2}{(39\times10^{-6})^2}}$

$C = 2.6 \times 10^{-9}$

$Z = L/CR = 39\times10^{-6} / 2.6 \times 10^{-9} \times 5$

$= 3 \times 10^3 \Omega$

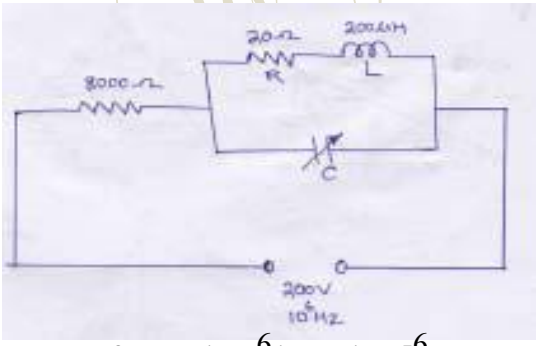
Example: A coil of resistance 20Ω and inductance of 200μH is in parallel with a variable capacitor. This combination is series with a resistor of 8000Ω.The voltage of the supply is 200V at a frequency of 10^6Hz.Calculate

i) the value of C to give resonance

ii) the Q of the coil

iii) the current in each branch of the circuit at resonance

Solution:



$X_L = 2\pi fL = 2\pi * 10^6 * 200 * 10^{-6} = 1256\Omega$

The coil is negligible resistance in comparison to reactance.

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$10^6 = \frac{1}{2\pi\sqrt{200 * C * 10^{-6}}}$$

ii) $Q=\frac{2\pi fL}{R} = 2\pi * 10^6 * 200 * \frac{10^{-4}}{20}$=62.8

iii) dynamic impedance of the circuit Z=L/CR=200*10^{-6}/(125*10^{-12}*20)=80000Ω

total Z=80000+8000=88000Ω

I=200/88000=2.27mA

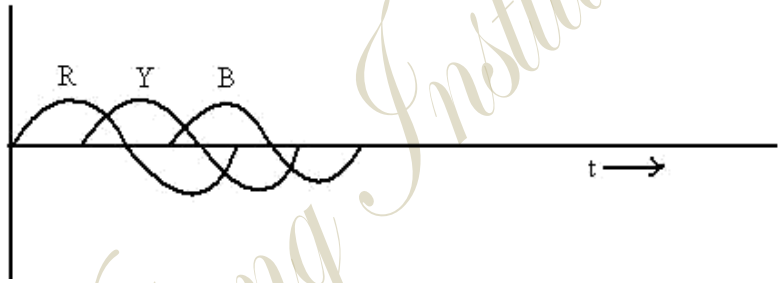
p.d across tuned circuit=2.27*10^{-3}*80000=181.6V current through inductive branch= $\frac{181.6}{\quad} = 144.5mA$

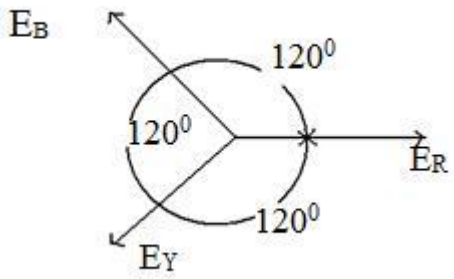
current through capacitor branch= ωVC

=181.6*2π*10^6*125*10^{-12}=142.7mA

POLY-PHASE CIRCUIT

Three-phase circuits consists of three windings i.e. R.Y.B





$E_R = E_m \sin wt = E_m \angle 0$

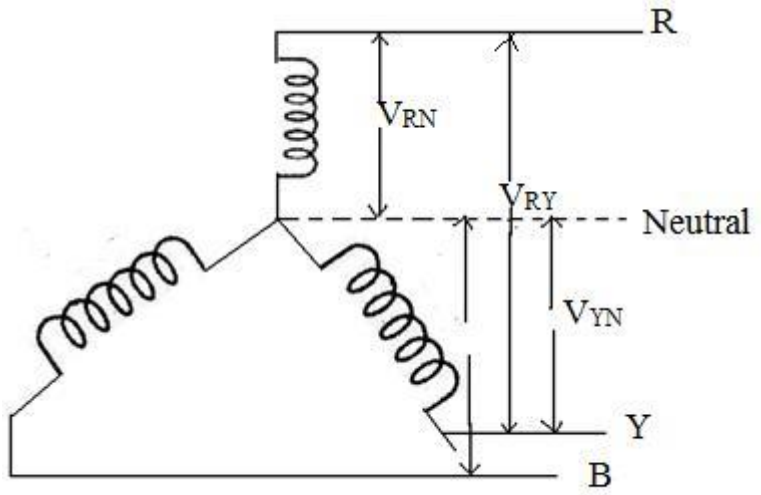
$E_Y = E_m \sin(wt - 120) = E_m \angle -120$

$E_B = E_m \sin(wt - 240) = E_m \angle -240 = E_m \angle 120$

3 - φ Circuit are divided into two types

- Star Connection
- Delta Connection

Star Connection :→



If three similar ends connected at one point, then it is known as star connected system.

The common point is known as neutral point and the wire taken from the neutral point is known as Neutral wire.

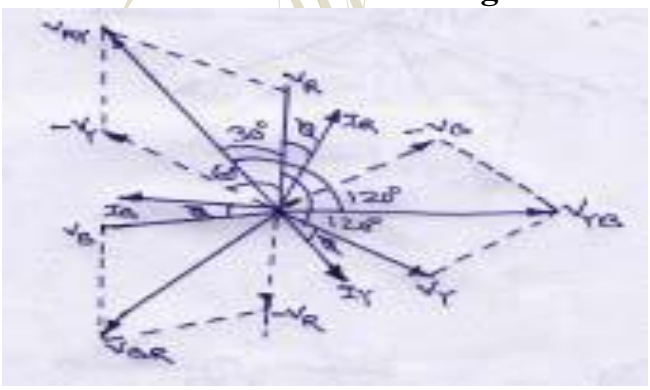
Phase Voltage :→

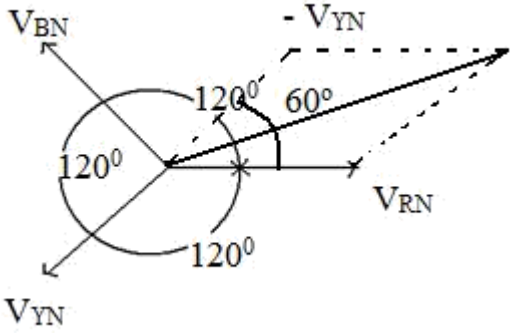
It is the potential difference between phase and Neutral.

Line Voltage : →

It is It is the potential difference between two phases.

Relation Between Phase Voltage and Line Voltage :→





Line Volatage $\vec{V}_{RY} = \vec{V}_{RN} - \vec{V}_{YN}$

$$V_L = \sqrt{V_{RN} + V_{YN} - 2V_{RN}V_{YN}\,Cos60^o}$$

$$= \sqrt{V^2_{Ph} + V^2_{ph} - 2V_{ph}V_{ph} \times \frac{1}{2}}$$

$$= \sqrt{3V^2_{Ph}} = \sqrt{3}V_{Ph}$$

$$V_L = \sqrt{3}V_{Ph}$$

Since in a balanced B –phase circuit $V_{RN} = V_{YN} = V_{BN} = V_{ph}$

Relation Between Line current and Phase Current :-

In case of star connection system the leads are connected in series with each phase

Hence the line current is equal to phase current

$I_L = I_{ph}$

Power in 3- Phase circuit:-

$P = V_{ph} I_{ph} \cos\varphi$ per phase

$= 3V_{ph} I_{ph} \cos\varphi$ for 3 phase

$= 3\frac{V_L}{\sqrt{3}} I_L \cos\varphi$ ($Q V_L = \sqrt{3}V_{ph}$

$P = \sqrt{3}V_L I_L \cos\varphi$

Summaries in star connection:

i) The line voltages are 120^0 apart from each other.

ii) Line voltages are 30^0 ahead of their respective phase voltage.

iii) The angle between line currents and the corresponding line voltage is 30+

iv) The current in line and phase are same.

Delta Connection :-

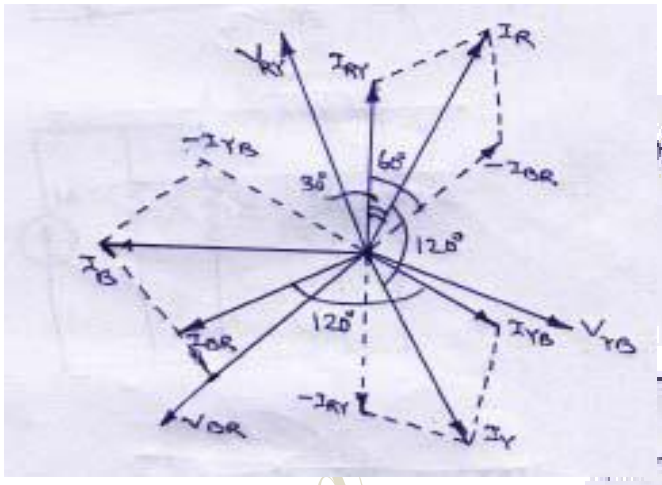
If the dissimilar ends of the closed mesh then it is called a Delta Connected system

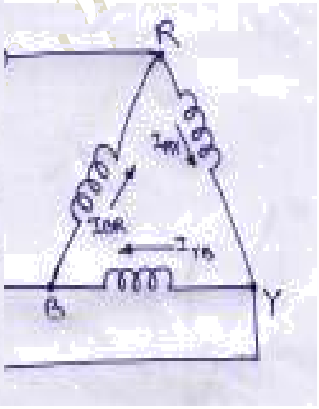
Relation Between Line Current and Phase Current :-

Line Current in wire – 1 $= i_{\vec{R}} - i_{\vec{Y}}$

Line Current in wire -2 $= i_{\vec{Y}} - i_{\vec{B}}$

Line Current in wire – 3 $= i_{\vec{B}} - i_{\vec{R}}$





$$I_L = \vec{I_R} - \vec{I_Y}$$

$$= \sqrt{I_R^2 + I_Y^2 - 2I_R I_Y \cos 60^0}$$

$$= \sqrt{I_{ph}^2 + I_{ph}^2 - 2I_{ph}I_{ph} \times \frac{1}{2}}$$

$$= \sqrt{3I_{ph}^2}\ ,\ I_L = \sqrt{3I_{ph}^2}$$

$$I_L = \sqrt{3} I_{ph}$$

Relation Between Line Voltage & Phase Voltage : →

$$V_L = V_{ph}$$

Power = $= \sqrt{3} V_L I_L \cos\varphi$

Summaries in delta:

i)Line currents are 120^o apart from each other.

ii) Line currents are 30^o behind the respective phase current.

iii) The angle between the line currents and corresponding line voltages is 30+

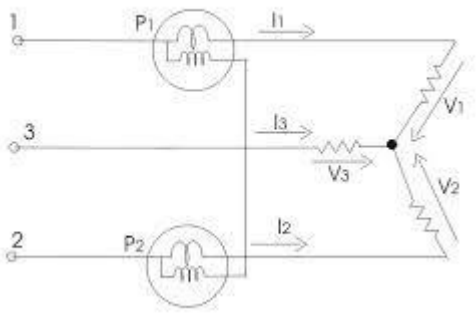
Measurement of Power : →

(1) By single watt-meter method

(2) By Two-watt meter Method

(3) By Three-watt meter Method

Measurement of power By Two Watt Meter Method :-



Phasor Diagram :-

Let V_R, V_Y,V_B are the r.m.s value of 3-φ voltages and I_R,I_Y,I_B are the r.m.s. values of the currents respectively.

Current in R-phase which flows through the current coil of watt-meter

$W_1=I_R$

And $W_2 = I_Y$

Potential difference across the voltage coil of $W_1 = \vec{V}_{RB} = \vec{V}_R - \vec{V}_B$

And $W_2 = \vec{V}_{YB} = \vec{V}_Y - \vec{V}_B$

Assuming the load is inductive type watt-meter W_1 reads.

$W_1 = V_{RB}\, I_R \cos(30 - \varphi)$ (1)

$W_1 = V_L\, I_L \cos(30 - \varphi)$ ----------

Wattmeter W_2 reads

$W_2 = V_{YB}\, I_Y \cos(30 + \varphi)$----------

$W_2 = V_L\, I_L \cos(30 + \varphi)$ (2)

$W_1 + W_2 = V_L\, I_L \cos(30 - \varphi) + V_L\, I_L \cos(30 + \varphi)$

$= V_L\, I_L [\cos(30 - \varphi) + V_L\, I_L \cos(30 + \varphi)]$

$= V_L\, I_L (2 \cos 30^o \cos\varphi)$

$= V_L I_L (2 \times \frac{\sqrt{3}}{2} \cos\varphi)$

$W_1 + W_2 = \sqrt{3} V_L\, I_L \cos\varphi$ (3)

$W_1 - W_2 = V_L\, I_L [\cos(30 - \varphi) - \cos(30 + \varphi)$

$$= V_L I_L (2\sin 30^o \sin \varphi)$$

$$= V_L I_L (2 \times \frac{1}{2} \times \sin \varphi)$$

$$W_1 - W_2 = V_L I_L \sin\varphi$$

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\varphi}{\sqrt{3} V_L I_L \cos\varphi}$$

$$\frac{1}{\sqrt{3}} = \tan\varphi$$

$$\tan\varphi = \sqrt{3}\frac{W_1 - W_2}{W_1 + W_2}$$

$$\Rightarrow \varphi = \tan^{-}\sqrt{\frac{W_1 - W_2}{3 W_1 + W_2}}$$

Variation in wattmeter reading with respect to p.f:

Pf	W_1 reading	W_2 reading
=0,cos =1	+ve equal	+ve equal
=60,cos =0.5	0	+ve
=90,cos =0	-ve, equal	+ve equal

Exp. : 01

A balanced star – connected load of (8+56). Per phase is connected to a balanced 3-phase 100-v supply. Find the cone current power factor, power and total volt-amperes.

Solution :

$$Z_{ph} = \sqrt{8^2 + 6^2} = 10\Omega$$

$$V_{ph} = 400 / \sqrt{3} = 23 / v$$

$$I_{ph} = V_{ph} / Z_{ph} = 231/10 = 23.1A$$

i) $I_L = Z_{ph} = 23.1A$

ii) P.f. = $\cos\theta = R_{ph}/z_{ph} = 8/10 = 0.8$ (lag)

iii) $Power P = \sqrt{3} V_L I_L \cos\theta$

$= \sqrt{3} \times 400 \times 23.1 \times 0.8$

= 12, 800 watt.

iv) Total volt ampere s $= \sqrt{3}\ V_L\ I_L$

$= \sqrt{3} \times 400 \times 23.1$

= 16, 000 VA.

Exp. -02

Phase voltage and current of a star-connected inductive load is 150V and 25A. Power factor of load as 0.707 (Lag). Assuming that the system is 3-wire and power is measured using two watt meters, find the readings of watt meters.

Solution :

$V_{ph} = 150V$

$V_L = \sqrt{3} \times 150$

$I_{ph} = I_L = 25A$

Total power = $\sqrt{3}\, V_L I_L \cos\varphi = \sqrt{3} \times 150 \times \sqrt{3} \times 25 \times 0.707 = 7954$ watt. $W_1 + W_2 = 7954.00$, $\cos\varphi = 0.707$

$\varphi = \cos^{-1}(0.707) = 45°$, $\tan 45° =$ 1 Now for a lagging power factor,

$$\tan\varphi = \sqrt{3}(W_1 - W_2)/(W_1 + W_2)$$

$$\Rightarrow 1 = \sqrt{3}\,\frac{(W_1 - W_2)}{7954}$$

$$\therefore (W_1 - W_2) = 4592w$$

From (i) and (ii) above, we get

$W_1 = 6273w$ $W_2 = 1681w$

TRANSIENTS

Whenever a network containing energy storage elements such as inductor or capacitor is switched from one condition to another,either by change in applied source or change in network elements,the response current and voltage change from one state to the other state.The time taken to change from an initial steady state to the final steady state is known as the *transient period*.This response is known as *transient response* or *transients.*The response of the network after it attains a final steady value is independent of time and is called the steady-state response.The complete response of the network is determined with the help of a differential equation.

STEADY STATE AND TRANSIENT RESPONSE

In a network containing energy storage elements, with change in excitation, the currents and voltages in the circuit change from one state to other state. The behaviour of the voltage or current when it is changed from one state to another is called the transient state. The time taken for the circuit to change from one steady state to another steady state is called the transient time. The application of KVL and KCL to circuits containing energy storage elements results in differential, rather than algebraic equations. when we consider a circuit containing storage elements which are independent of the sources, the response depends upon the nature of the circuit and is called natural response. Storage elements deliver their energy to the resistances. Hence, the response changes, gets saturated after some time,and is referred to as the transient response. When we consider a source acting on a circuit, the response depends on the nature of the source or sources.This response is called forced response. In other words,the complete response of a circuit consists of two parts; the forced response and the transient response. When we consider a differential equation, the complete solution consists of two parts: the complementary function and the particular solution. The complementary function dies out after short interval, and is referred to as the transient response or source free response. The particular solution is the steady state response, or the forced response. The first step in finding the complete solution of a circuit is to form a differential equation for the circuit. By obtaining the differential equation, several methods can be used to find out the complete solution.

DC RESPONSE OF AN R-L CIRCUIT

Consider a circuit consisting of a resistance and inductance as shown in figure.The inductor in the circuit is initially uncharged and is in series with the resistor.When the switch S is closed ,we can find the complete solution for the current.Application of kirchoff's voltage law to the circuit results in the following differential equation.

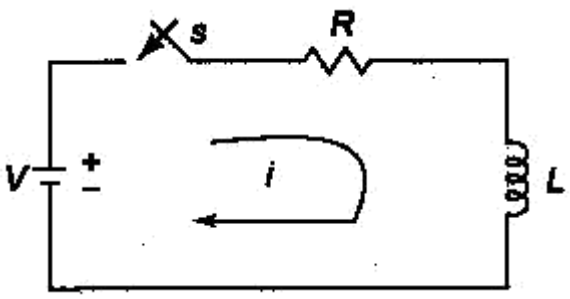


Figure 1.1

$V = Ri + L\frac{di}{dt}$ 1.1

Or $\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L}$ 1.2

In the above equation , the current I is the solution to be found and V is the applied constant voltage. The voltage V is applied to the circuit only when the switch S is closed. The above equation is a linear differential equation of first order.comparing it with a non-homogenious differential equation

$\frac{dx}{dt} + P x = K$ 1.3

whose solution is

$x = e^{-Pt} \int K e^{Pt}$ 1.4

Where c is an arbitrary constant. In a similar way , we can write the current equation as

$i = c\ e^{-\left(\frac{R}{L}\right)t} + e^{-\left(\frac{R}{L}\right)t} \int \frac{V}{L} e^{\left(\frac{R}{L}\right)t} dt$

Hence , $i = c\ e^{-\left(\frac{R}{L}\right)t} + \frac{V}{R}$ 1.5

To determine the value of c in equation c , we use the initial conditions .In the circuit shown in Fig.1.1, the switch s is closed at t=0.at t=0-,i.e. just before closing the switch s , the current in the inductor is zero. Since the inductor does not allow sudden changes in currents, at t=o+ just after the switch is closed,the current remains zero.

Thus at t = 0, i =0

Substituting the above condition in equation c , we have

$0 = c + \frac{V}{R}$

Substituting the value of c in equation c , we get

$i = \frac{V}{R} - \frac{V}{R} e^{\frac{-Rt}{L}}$

$i = \frac{V}{R} (1 - e^{\frac{-Rt}{L}})$

$i = I_o (1 - e^{\frac{-Rt}{L}})$ (where $I_o = \frac{V}{R}$)

$i = I_o (1 - e^{\frac{-t}{\tau}})$ (where $\tau = Time\ constant = \frac{L}{R}$) 1.6

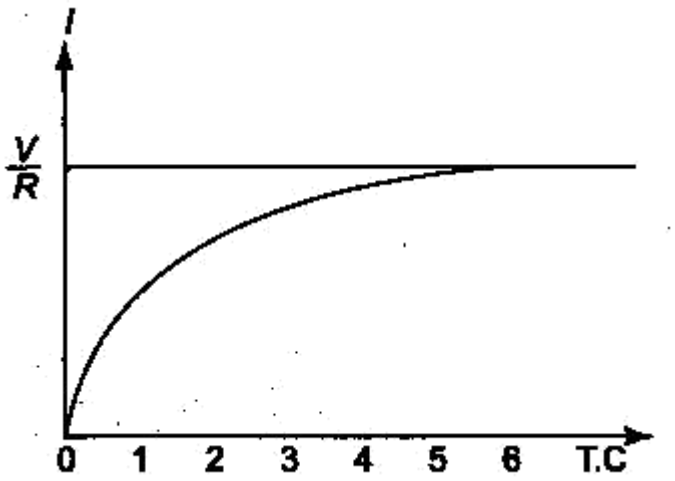


Figure 1.2

Equation d consists of two parts, the steady state part (I_0 = V/R) and the transient part $I_{-} e^{\frac{-Rt}{L}}$.

When switch S is closed , the response reaches a steady state value after a time interval as shown in figure 1.2.

Here the transition period is defined as the time taken for the current to reach its final or stedy state value from its initial value.In the transient part of the solution, the quantity L/R is important in describing the curve since L/R is the time period required for the current to reach its initial value of zero to the final value I_{-}=V/R. The time constant of a function $I_{-} e^{\frac{-Rt}{L}}$ is the time at which the exponent of e is unity, where e is the base of the natural logarithms.The term L/R is called the time constant and is denoted by τ .

So, $\tau = \frac{L}{R}$ sec

Hence, the transient part of the solution is

$$i = -\frac{V}{R} e^{\frac{-Rt}{L}} = \frac{V}{R} e^{\frac{-t}{\tau}}$$

At one Time constant , the transient term reaches 36.8 percent of its initial value.

$$i(\tau) = -\frac{V}{R} e^{\frac{-\tau}{\tau}} = -\frac{V}{R} e^{-1} = -0.368 \frac{V}{R}$$

Similarly,

$$i(2\tau) = -\frac{V}{R} e^{-2} = -0.135 \frac{V}{R}$$

$$i(3\tau) = -\frac{V}{R} e^{-3} = -0.0498 \frac{V}{R}$$

$$i(5\tau) = -\frac{V}{R} e^{-5} = -0.0067 \frac{V}{R}$$

After 5 TC the transient part reaches more than 99 percent of its final value.

In figure A we can find out the voltages and powers across each element by using the current. Voltage across the resistor is

$$v_R = R\,i = R \times \frac{V}{R}\left(1 - e^{\frac{-Rt}{L}}\right)$$

Hence, $v_R = V\left(1 - e^{\frac{-Rt}{L}}\right)$

Similarly, the voltage across the inductance is

$$v_l = L\frac{di}{dt} = L\,\frac{V}{L} \times \frac{R}{L}\, e^{\frac{-Rt}{L}} = V\, e^{\frac{-Rt}{L}}$$

The responses are shown in Figure 1.3.

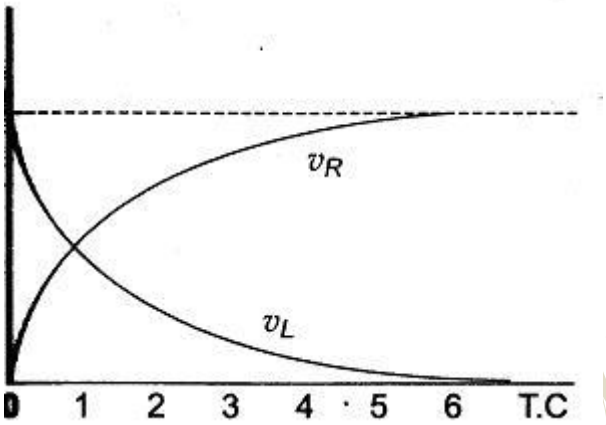


Figure 1.3

Power in the resistor is

$$P_R = v_R\, i = V\left(1 - e^{\frac{-Rt}{L}}\right)\left(1 - e^{\frac{-Rt}{L}}\right) \times \frac{V}{R}$$

$$= \frac{V^2}{R}\left(1 - 2e^{\frac{-Rt}{L}}\right) + e^{\frac{-2Rt}{L}}$$

Power in the inductor is

$$P_l = v_l\, i = V\, e^{\frac{-Rt}{L}} \times \frac{V}{R}\left(1 - e^{\frac{-Rt}{L}}\right)$$

$$= \frac{V^2}{R}\left(e^{\frac{-Rt}{L}} - e^{\frac{-2Rt}{L}}\right)$$

The responses are shown in figure 1.4 .

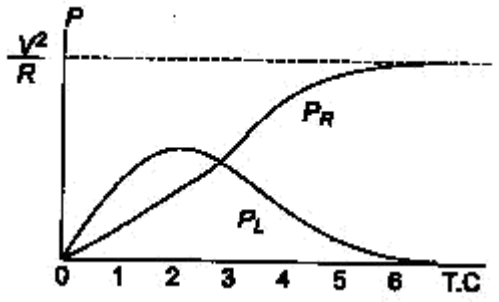


Figure 1.4

Problem : 1.1

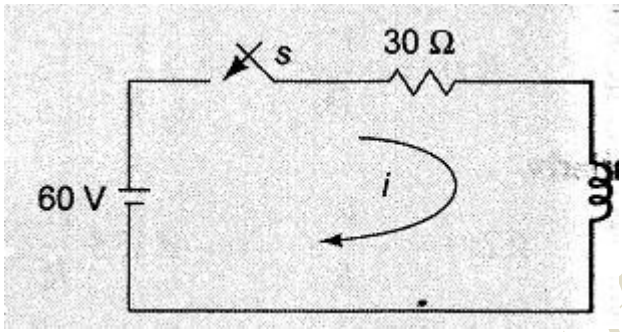


Figure 1.5

A series R-L circuit with R = 30Ω and L = 15 H has a constant voltage V = 50 V applied at t=0 as shown in Fig. 1.5 . determine the current i, the voltage across resistor and across inductor.

Solution :

By applying Kirchoff's voltage Law, we get

$15\frac{di}{dt}+30i=60$

$\frac{di}{dt}+2i=4$

The general solution for a linear differential equation is

$i=ce^{-\int p\,dt}+e^{-pt}\int Ke^{pt}dt$

where P=2,K=4

putting the values

$i=ce^{-2t}+e^{-2t}\int 4e^{2t}dt$

$i=ce^{-2t}+2$

At t=0, the switch s is closed.

Since the inductor never allows sudden change in currents. At t= 0^- the current in the circuit is zero. Therefore at t= 0^+, i =0

$0 = c + 2$

$c = -2$

Substituting the value of c in the current equation, we have

$i = 2(1 - e^{-2t})$ A

voltage across resistor (V_R) =iR =$2(1 - e^{-2t}) \times 30 = 60(1 - e^{-2t})$ v

voltage across inductor (V_L) = $L\frac{di}{dt}$ = $15 \times \frac{d}{dt} 2(1 - e^{-2t}) = 30 \times 2e^{-2t}$ v= $60e^{-2t}$

DC RESPONSE OF AN R-C CIRCUIT

Consider a circuit consisting of a resistance and capacitance as shown in figure.The capacitor in the circuit is initially uncharged and is in series with the resistor.When the switch S is closed at t=0 , we can find the complete solution for the current.Application of kirchoff's voltage law to the circuit results in the following differential equation.

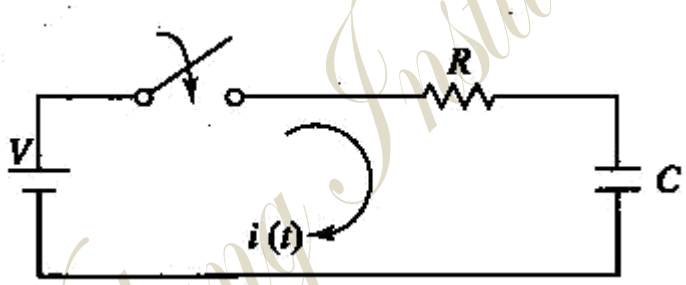


Figure 1.6

$$V = Ri + \frac{1}{C}\int i\,dt \quad \text{.....1.7}$$

By differentiating the above equation, we get

$$0 = R\frac{di}{dt} + \frac{i}{C} \quad \text{.....1.8}$$

Or

$$\frac{di}{dt} + \frac{1}{RC} i = 0 \quad \text{.....1.9}$$

Equation c is a linear differential equation with only the complementary function. The particular solution for the above equation is zero. The solution for this type of differential equation is

$$i = c\, e^{-\left(\frac{t}{RC}\right)} \quad \text{.....1.10}$$

To determine the value of c in equation c , we use the initial conditions .In the circuit shown in Fig. the switch s is closed at t=0. Since the capacitor does not allow sudden changes in voltage, it will act as a short circuit at t=o+ just after the switch is closed.

So the current in the circuit at t = 0+ is $\frac{V}{R}$

Thus at t = 0, the current i = $\frac{V}{R}$

Substituting the above condition in equation c , we have

$$\frac{V}{R} = c$$

Substituting the value of c in equation c , we get

$$i = \frac{V}{R} e^{-\left(\frac{t}{RC}\right)} \quad \text{.....1.11}$$

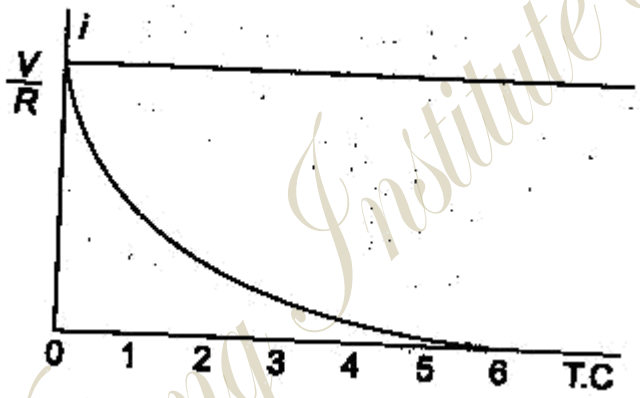


Figure 1.7

When switch S is closed , the response decays as shown in figurre.

The term RC is called the time constant and is denoted by τ .

So, τ = RC sec

After 5 TC the curve reaches 99 percent of its final value.

In figure A we can find out the voltage across each element by using the current equation.

Voltage across the resistor is

v_R =R i = R $\times \frac{V}{R} e^{\frac{-t}{RC}}$

Hence , v_R=V $e^{\frac{-t}{RC}}$

Similarly, voltage across the capacitor is

$v_c = \frac{1}{C}\int i\, dt$

$= \frac{1}{C}\int \frac{V}{R} e^{\frac{-t}{RC}} dt$

$= -\left(\frac{V}{RC} \times RC\, e^{\frac{-t}{RC}}\right) + c$

$= -V\, e^{\frac{-t}{RC}} + c$

At t=0,voltage across capacitor is zero

So, c = V

And

$V_c = V\left(1 - e^{\frac{-t}{RC}}\right)$

The responses are shown in Figure1.8.

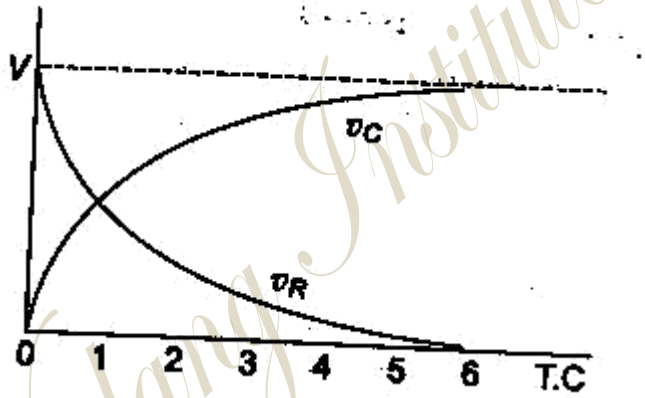


Figure 1.8

Power in the resistor is

$P_R = v_R$ i = V $e^{\frac{-t}{RC}} \times \frac{V}{R} e^{\frac{-t}{RC}}$

$= \frac{V^2}{R} e^{\frac{-2t}{RC}}$

Power in the capacitor is

$P_c = v_c$ i = V (1- $e^{\frac{-t}{RC}}$) $\frac{V}{R} e^{\frac{-t}{RC}}$

$= \frac{V^2}{R}\left(e^{\frac{-t}{RC}} - e^{\frac{-2t}{RC}}\right)$

The responses are shown in figure 1.9.

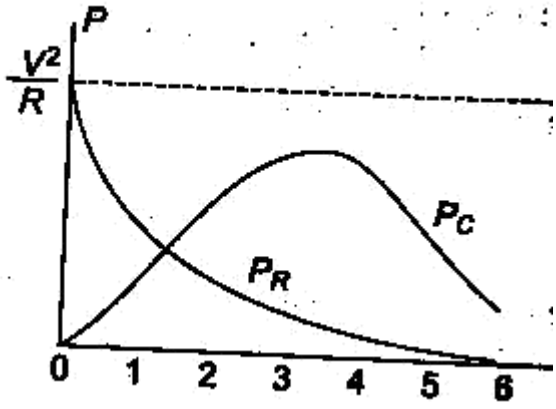


Figure 1.9

Problem : 1.2

A series R-C circuit with R = 10Ω and C =0.1 F has a constant voltage V = 20 V applied at t=0 as shown in Fig. determine the current i, the voltage across resistor and across capacitor.

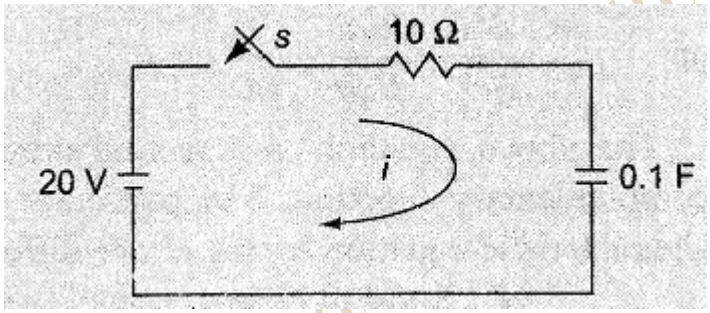


Figure 1.10

Solution :

By applying Kirchoff's voltage Law, we get

$10i + \frac{1}{0.1}\int i\,dt = 20$

Differentiating w.r.t. t we get

$10\frac{di}{dt} + \frac{i}{0.1} = 0$

$= 10\frac{di}{dt} + i = 0$

The solution for above equation is

i=c

At t=0, the switch s is closed.

Since the capacitor never allows sudden change in voltages. At t= the current in the circuit is i = V/R=20/10 =2 A

. Therefore at t= 0, i =2 A

= the current equation is i=2

voltage across resistor ( ) =iR =2 x 10=20 v

voltage across capacitor ( ) = V = 20(1-) V

DC RESPONSE OF AN R-L-C CIRCUIT

Consider a circuit consisting of a resistance, inductance and capacitance as shown in figure.The capacitor and inductor in the circuit is initially uncharged and are in series with the resistor.When the switch S is closed at t=0 , we can find the complete solution for the current.Application of kirchoff's voltage law to the circuit results in the following differential equation.

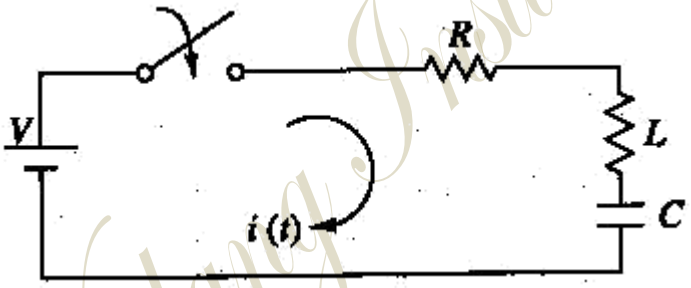


Figure 1.11

V = Ri + L $\frac{di}{dt}$ + $\frac{1}{c}\int t\,dt$ 1.12

By differentiating the above equation, we get

0=R $\frac{dt}{dt}$ $L\,d^2t/dt^2 + \frac{i}{C}$ i = ...1.13

Or

$d^2t/dt^2 + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}$ i =0 .....1.14

The above equation c is a second order linear differential equation with only the complementary function. The particular solution for the above equation is zero. The characteristics equation for this type of differential equation is

$D^2 + \frac{R}{L} D + \frac{1}{LC} = 0$ 1.15

The roots of equation 1.15 are

$D_1, D_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

By assuming $K_1 = \frac{R}{2L}$ and $K_2 = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

$D_1 = K_1 + K_2$ and $D_2 = K_1 - K_2$

Here K_2 may be positive,negative or zero .

Case I : K_2 *is Positive* $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

Then , the roots are Real and Unequal and give an over damped Response as shown in figure 1.12.

The solution for the above equation is : $i = C_1 e^{(K_1+K_2)t} + C_2 e^{(K_1-K_2)t}$

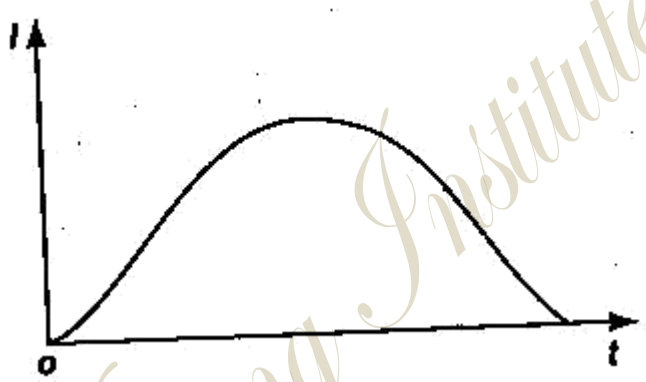


Figure 1.12

Case II : K_2 *is Negative* $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

Then , the roots are Complex Conjugate, and give an under-damped Response as shown in figure 1.13.

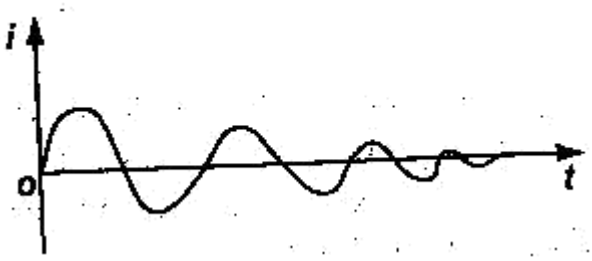


Figure 1.13

The solution for the above equation is : i = $e^{K_2 t}(C_1 \cos K_2 t + C_2 \sin K_2 t)$

Case III : K_2 *is Zero* $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

Then , the roots are Equal and give an Critically-damped Response as shown in figure 1.14.

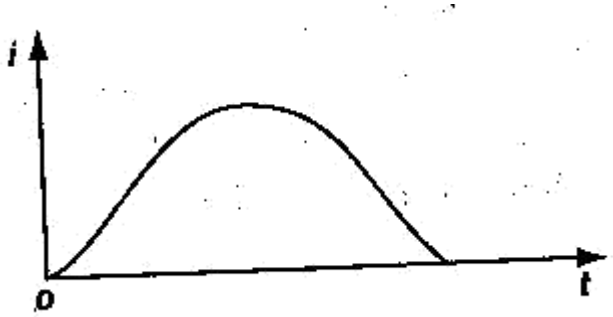


Figure 1.14

The solution for the above equation is : i = $e^{K_2 t}(C_1 + C_2 t)$

Problem : 1.3

A series R-L-C circuit with R = 20Ω , L = 0.05H and C = 20 μF has a constant voltage V = 100 V applied at t=0 as shown in Fig. determine the transient current i .

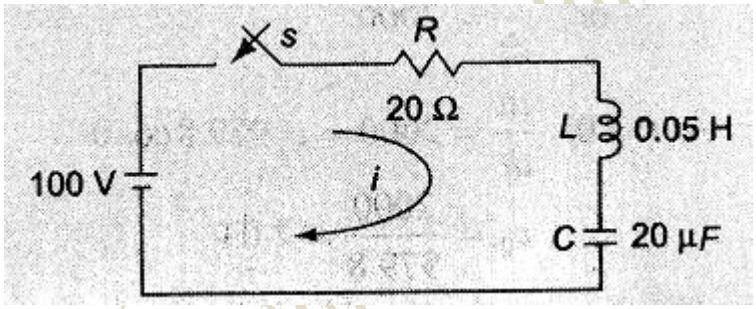


Figure 1.15

Solution :

By applying Kirchoff's voltage Law, we get

$100=30i+0.05\frac{di}{dt}+\frac{1}{20\times10^{-6}}\int i\,dt$

Differentiating w.r.t. t we get

$0.05d^2i/dt^2+20\ \frac{di}{dt}+\ \frac{1}{20\times10^{-6}}i=0$

$\Rightarrow d^2i/dt^2 + 400\frac{di}{dt} + 10^6 i = 0$

$\Rightarrow (D^2 + 400D + 10^6) i = 0$

The roots of equation are

$D_1, D_2 = -\frac{400}{2} \pm \sqrt{\left(\frac{400}{2}\right)^2 - 10^6}$

$= -200 \pm \sqrt{(200)^2 - 10^6}$

$D_1 =$ -200+j979.8

$D_2 =$ -200-j979.8

$i = e^{-200t}[C_1 \cos K_2 t + C_2 \cos K_2 t]$

$i = e^{-200t}[C_1 \cos 979.8t + C_2 \sin 979.8t]$ A

At t=0, the switch s is closed.

Since the inductor never allows sudden change in currents. At t= 0^- the current in the circuit is zero. Therefore at t= 0^-, i =0

$\Rightarrow i = 0 = (1)\ [C_1 \cos 0 + C_2 \sin 0]$

$\Rightarrow C_1 = 0$ and $i = e^{-200t}[C_2 \sin 979.8t]$ A

Differentiating w.r.t. t we get

$$\frac{di}{dt} = C_2[e^{-200t} 979.8 \cos 979.8t + e^{-200t}(-200)\sin 979.8t]$$

At t=0, the voltage across the inductor is 100 V

$\Rightarrow L\frac{di}{dt} = 100$ or $\frac{di}{dt} = 2000$

At t=0, $\frac{di}{dt} = 2000 = C_2 979.8 \cos 0$

$\Rightarrow C_2 = \frac{2000}{979.8} = 2.04$

The current equation is

i= $e^{-200t}(2.04\sin 979.8t)\,A$

ANALYSIS OF CIRCUITS USING LAPLACE TRANSFORM TECHNIQUE

The Laplace transform is a powerful Analytical Technique that is widely used to study the behavior of Linear,Lumped parameter circuits. Laplace Transform converts a time domain function f(t) to a frequency domain function F(s) and also Inverse Laplace transformation converts the frequency domain function F(s) back to a time domain function f(t).

L { f(t)} = F(s) = $\int_{-\infty}^{\infty} e^{-st}$ f(t) dt ..... LT 1

ds ..... LT 2

DC RESPONSE OF AN R-L CIRCUIT (LT Method)

Let us determine the solution i of the first order differential equation given by equation A which is for the DC response of a R-L Circuit under the zero initial condition i.e. current is zero, i=0 at t= 0^- and hence i=0 at t= 0^+ in the circuit in figure A by the property of Inductance not allowing the current to change as switch is closed at t=0.

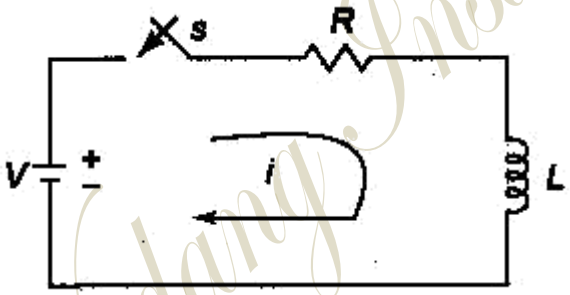


Figure LT 1.1

V = Ri + L $\frac{di}{dt}$ .....LT 1.1

Taking the Laplace Transform of bothe sides we get,

= R I(s) + L [s I(s) –I(0)] ..... LT 1.2

=R I(s) + L [s I(s)] (I(0) =0 : zero initial current)

= I(s)[R +L s]

S

$\Rightarrow \frac{V}{S}$

$\Rightarrow \frac{V}{S}$

$\Rightarrow I(s) = \frac{V}{s[R+Ls]}$ ..... LT 1.3

Taking the Laplace Inverse Transform of both sides we get,

$\Rightarrow L^{-1}\{I(s)\} = i(t) = L^{-1}\{\frac{V}{s[R+Ls]}\}$

i(t) = $L^{-1}\{\frac{V/L}{s[R/L+s]}\}$ (Dividing the numerator and denominator by L)

putting $\alpha = R/L$ we get

i(t) = $L^{-1}\{\frac{V/L}{s[s+\alpha]}\} = L^{-1}\{\frac{V}{L}(\frac{1}{s}-\frac{1}{(s+\alpha)})\frac{1}{\alpha}\}$

i(t) = $L^{-1}\{\frac{V}{L}(\frac{1}{s}-\frac{1}{(s+R/L)})\frac{L}{R}\}$ (again putting back the value of α)

i(t) = $L^{-1}\{\frac{V}{R}(\frac{1}{s}-\frac{1}{(s+R/L)})\} = \frac{V}{R}(1- e^{\frac{-Rt}{L}}) = I_o(1- e^{\frac{-Rt}{L}})$ (where $I_o = \frac{V}{R}$)

i(t)= I_o (1-$e^{\frac{-t}{\tau}}$) (where $\tau = Time\ constant = \frac{L}{R}$) .....LT 1.4

It can be observed that solution for i(t) as obtained by Laplace Transform technique is same as that obtained by standard differential method .

DC RESPONSE OF AN R-C CIRCUIT(L.T.Method)

Similarly ,

Let us determine the solution i of the first order differential equation given by equation A which is for the DC response of a R-C Circuit under the zero initial condition i.e. voltage across capacitor is zero, V_c =0 at t= 0_- and hence V_c =0 at t= 0_+ in the circuit in figure A by the property of capacitance not allowing the voltage across it to change as switch is closed at t=0.

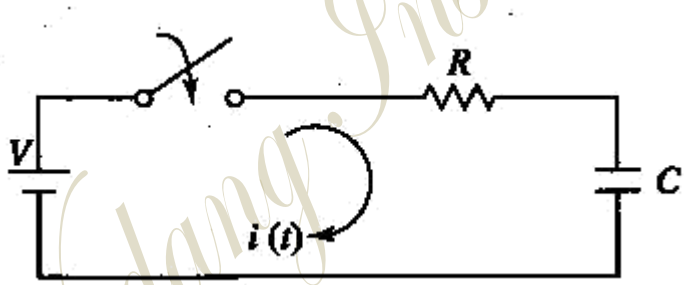


Figure LT 1.2

V = Ri + $\frac{1}{C}\int i\,dt$ .....LT 1.5

Taking the Laplace Transform of both sides we get,

$\frac{V}{S}$ =R I(s) + $\frac{1}{C}$[$\frac{I(s)}{s}$ +I (0)] .....LT 1.6

$\Rightarrow \frac{V}{S}$ =R I(s) + $\frac{1}{C}$[$\frac{I(s)}{s}$] (I(0) =0 : zero initial charge)

$\Rightarrow \frac{V}{S}$ = I(s)[R + $\frac{1}{Cs}$] = I(s)[$\frac{Rcs+1}{Cs}$]

$\Rightarrow I(s) = \frac{V}{s}\left[\frac{Cs}{(RCs+1)}\right] = \frac{VC}{(RCs+1)}$ .....LT 1.7

Taking the Laplace Inverse Transform of both sides we

get, $\Rightarrow L^{-1}\{I(s)\} = i(t) = L^{-1}\left\{\frac{VC}{(RCs+1)}\right\}$

$i(t) = L^{-1}\left\{\frac{\frac{VC}{RC}}{\left[s+\frac{1}{RC}\right]}\right\}$ (Dividing the numerator and denominator by RC)

putting $\alpha = \frac{1}{RC}$ we get

$i(t) = L^{-1}\left\{\frac{V/R}{[s+\alpha]}\right\} = \frac{V}{R}e^{-\alpha t}$

$i(t) = \frac{V}{R}e^{\frac{-t}{RC}}$ (putting back the value of α)

$i(t) = I_o e^{\frac{-t}{RC}}$ (where $I_o = \frac{V}{R}$) .....LT 1.8

$i(t) = I_o e^{\frac{-t}{\tau}}$) (where $\tau = Time\ constant =$ RC)

It can be observed that solution for i(t) as obtained by Laplace Transform technique in q is same as that obtained by standard differential method in d.

DC RESPONSE OF AN R-L-C CIRCUIT (L.T. Method)

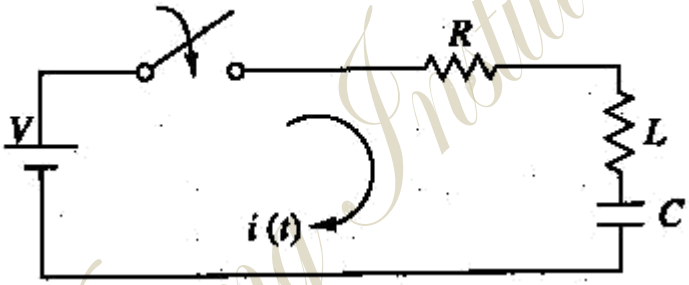


Figure LT 1.3

Similarly ,

Let us determine the solution i of the first order differential equation given by equation A which is for the DC response of a R-L-C Circuit under the zero initial condition i.e. the switch s is closed at t=0.at t=0-,i.e. just before closing the switch s , the current in the inductor is zero. Since the inductor does not allow sudden changes in currents, at t=o+ just after the switch is closed,the current remains zero. also the voltage across capacitor is zero i.e. V_c =0 at t= 0^- and hence V_c =0 at t= 0^+ in the circuit in figure by the property of capacitance not allowing the voltage across it V_c to suddenly change as switch is closed at t=0.

$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$ .....LT 1.9

Taking the Laplace Transform of both sides we get,

$\frac{V}{S}$ =R I(s) ++ L [s I(s) –I(0)]+$\frac{1}{C}$ [$\frac{I(s)}{s}$ +I (0)] ……LT 1.10

$=>\frac{V}{S}$ =R I(s) + L [s I(s)] + $\frac{1}{C}$ [$\frac{I(s)}{s}$] (I(0) = 0: zero initial current & I(0) =0 : zero initial charge)

$=>\frac{V}{S}$ = I(s)[R +Ls + $\frac{1}{Cs}$] = I(s)[$\frac{LCs^2+RCs+1}{Cs}$]

=>I(s) = $\frac{V}{s}$ [$\frac{Cs}{(LCs^2+RCs+1)}$] =$\frac{VC}{(LCs^2+RCs+1)}$ ……LT 1.11

Taking the Laplace Inverse Transform of both sides we get,

$=> L^{-1}\{I(s)\} = i(t) = L^{-1}\{\frac{VC}{(LCs^2+RCs+1)}\}$

i(t) = $L^{-1}\{\frac{\frac{VC}{LC}}{[s^2+s\frac{R}{L}+\frac{1}{LC}]}\}$ (Dividing the numerator and denominator by LC)

i(t) = $L^{-1}\{\frac{\frac{V}{L}}{[s^2+s\frac{R}{L}+\frac{1}{LC}]}\}$

putting $\propto = \frac{R}{2L}$ and $\omega = \sqrt{\frac{1}{LC}}$ we get

i(t) = $L^{-1}\{\frac{\frac{V}{L}}{[s^2+2\propto s+\omega^2]}\}$

where, $s_1, s_2 = \frac{-2\propto \pm \sqrt{4\propto^2-4\omega^2}}{2} = -\propto \pm \sqrt{\propto^2-\omega^2} = -\propto \pm \beta$

where, $\propto = \frac{R}{2L}$ $\omega = \sqrt{\frac{1}{LC}}$ and $\beta = \sqrt{\propto^2-\omega^2}$

By partial Fraction expansion , of I(s) ,

$$I(s) = \frac{A}{s-s_1} + \frac{B}{s-s_1}$$

$$= \frac{\frac{V}{L}}{(s_1-s_2)}$$

$$B = (s-s_2)\, I(s)]_{s=s_2}$$

$$= \frac{\frac{V}{L}}{(s_2-s_1)} = -\frac{\frac{V}{L}}{(s_1-s_2)}$$

$$I(s) = \frac{\frac{V}{L}}{(s_1-s_2)}\left(\frac{1}{(s-s_1)} - \frac{1}{(s-s_2)}\right)$$

Taking the Inverse Laplace Transform

$$i(t) = \ldots e^{s_1 t} + A_2 e^{s_2 t}$$

Where ... and ... are constants to be determined and ... and ... aren the roots of the equation.

Now depending upon the values of ... and ... , we have three cases of the response.

CASE I : When the roots are Real and Unequal, it gives an over-damped response.

or ... ; In this case, the solution is given by

$$i(t) = \ldots (A_1 e^{\beta t} \ldots A_2 e^{-\beta t}) \text{ ....LT 1.12}$$

or $\quad i(t) = \ldots e^{s_1 t} \ldots A_2 e^{s_2 t} \ldots$

CASE II : When the roots are Real and Equal, it gives an Critically-damped response.

$\frac{R}{2L} = \sqrt{\frac{1}{LC}}$ or ... ; In this case, the solution is given by

or

$i(t) = e^{-\alpha t} \ldots A_2 t \ldots)$ for t ... 0 ....LT 1.13

CASE III : When the roots are Complex Conjugate, it gives an under-damped response.

$\frac{R}{2L} < \sqrt{\frac{1}{LC}}$ or $\alpha < \omega$; In this case, the solution is given by

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad \text{for } t > 0$$

where, $s_1, s_2 = \frac{-2\alpha \pm \sqrt{4\alpha^2 - 4\omega^2}}{2} = -\alpha \pm \sqrt{\alpha^2 - \omega^2}$

Let $\sqrt{\alpha^2 - \omega^2} = \sqrt{-1}\sqrt{\omega^2 - \alpha^2} = j\,\omega_d$ where $j = \sqrt{-1}$ and $\omega_d = \sqrt{\omega^2 - \alpha^2}$

Hence , $\quad i(t) = e^{-\alpha t}(A_1 e^{j\omega_d t} + A_2 e^{-j\omega_d t})$

$$i(t) = e^{-\alpha t}\left[(A_1 + A_2)\left\{\frac{e^{j\omega_d t} + e^{-j\omega_d t}}{2}\right\} + j(A_1 - A_2)\left\{\frac{e^{j\omega_d t} - e^{-j\omega_d t}}{2j}\right\}\right]$$

$$i(t) = e^{-\alpha t}\left[(A_1 + A_2)\cos\omega_d t + j(A_1 - A_2)\sin\omega_d t\right]$$

$$i(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t) \text{ ....LT 1.14}$$

,,xxxxxxxxxxxxxxxxxx,,

TWO PORT NETWORKS

Generally, any network may be represented schematically by a rectangular box. A network may be used for representing either Source or Load , or for a variety of purposes. A pair of terminals at which a signal may enter or leave a network is called a port. A port is defined as any pair of terminals into which energy is withdrawn ,or where the network variables may be measured .One such network having only one pair of terminals (1-1')is shown figure 1.1.

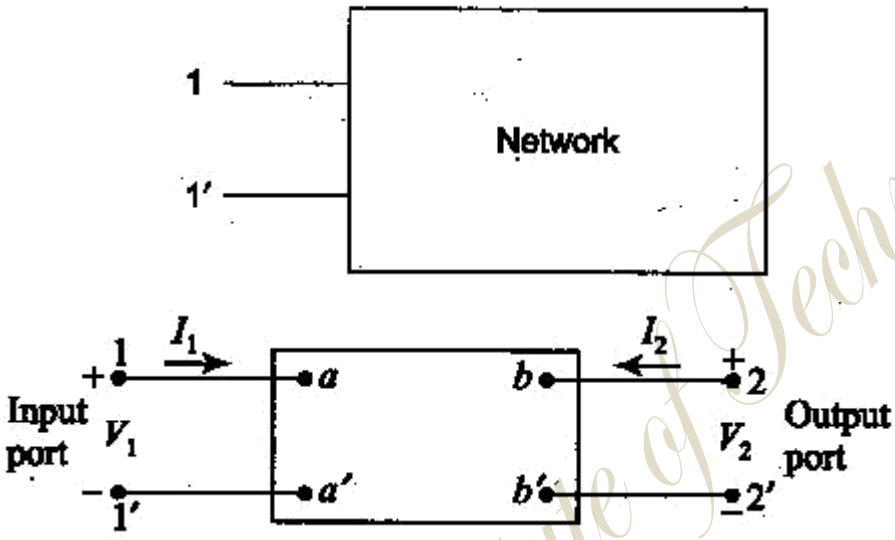


Figure 1.1

A two-port network is simply a network a network inside a black box, and the network has only two pairs of accessible terminals; usually one one pairs represents the input and the other represents the output. Such a building block is very common in electronic systems, communication system, transmission and distribution system. fig 1.1 shows a two-port network,or two terminal pair network,in which the four terminals have been paired into ports 1-1' and 2-2'.The terminals 1-1' together constitute a port. Similarly, the terminals 2-2' constitute another port. Two ports containing no sources in their branches are called passive ports ; among them are power transmission lines and transformers. Two ports containing source in their branches are called active ports. A voltage and current assigned to each of the two ports. The voltage and current at the input terminals are V_1 and I_1 ; where as V_2 and I_2 are entering into the network are V_1 , V_2,and I_1, I_2. Two of these are dependent variable, the other two are indepent variable. The number of possible combinations generated by four variable, taken two at time, is six. Thus, there are six possible sets of equations describing a two-port network.

OPEN CIRCUIT IMPEDANCE (Z) PARAMETERS

A general linear two-port network is shown below in figure 1.2.

The z parameters of a two-port network for the positive direction of voltages and currents may be defined by expressing the port voltages V_1 and V_2 in terms of the currents I_1 and I_2. Here V_1 and V_2 are two dependent variables and I_1 and I_2 are two independent variables.

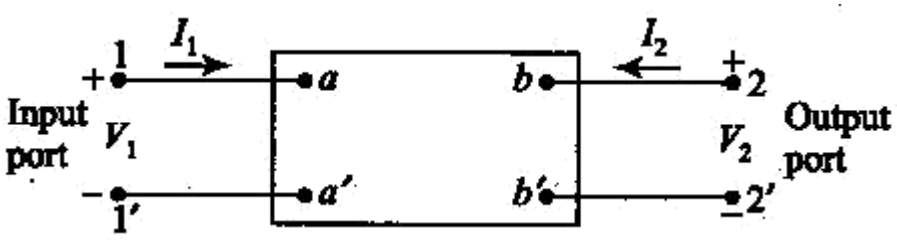


Figure 1.2

The voltage at port 1-1' is the response produced by the two currents I_1 and I_2.

thus

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad \text{..... 1.1}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad \text{..... 1.2}$$

Z_{11}, Z_{12}, Z_{21} and Z_{22} are the network functions, and are called impedance(Z) parameters, and are defined by equations 1.1 and 1.2 .

These parameters also can be represented by Matrices .

We may write the matrix equation [V] = [Z][I]

where V is the column matrix = $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

Z is a square matrix = $\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

and we may write $[I]$ in the column matrix = = $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

Thus, $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

The individual Z parameters for a given network can be defined by setting each of the port currents equal to zero. suppose port 2-2' is left open circuited, then I_2 =0.

Thus $Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0}$

where
Z_{11} is the driving point impedance at port 1 – 1' with port 2 – 2' open circuited. It is called the open circuit input impedance.

similarly,

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0}$$

where
Z_{21} is the transfer impedance at port 1 – 1' with port 2 – 2' open circuited. It is called the open circuit forward transfer impedance

.

Suppose port 1-1' is left open circuited, then I_2 =0.

Thus, $Z_{12} = \left.\frac{V_1}{I_2}\right| I_1 = 0$

where

Z_{12} is the transfer impedance at port $2-2'$ with port $1-1'$ open circuited. It is called the open circuit reverse transfer impedance

.

similarly,

$$Z_{22} = \left.\frac{V_2}{I_2}\right| I_1 = 0$$

where

Z_{22} is the open circuitdriving point impedance at port $2-2'$ with port $1-1'$ open circuited. It is also called the open circuit output impedance

.The equivalent circuit of the two-port networks governed by the equations 1.1 and 1.2 ,i.e. open circuit impedance parameters as shown below in fig 1.3.

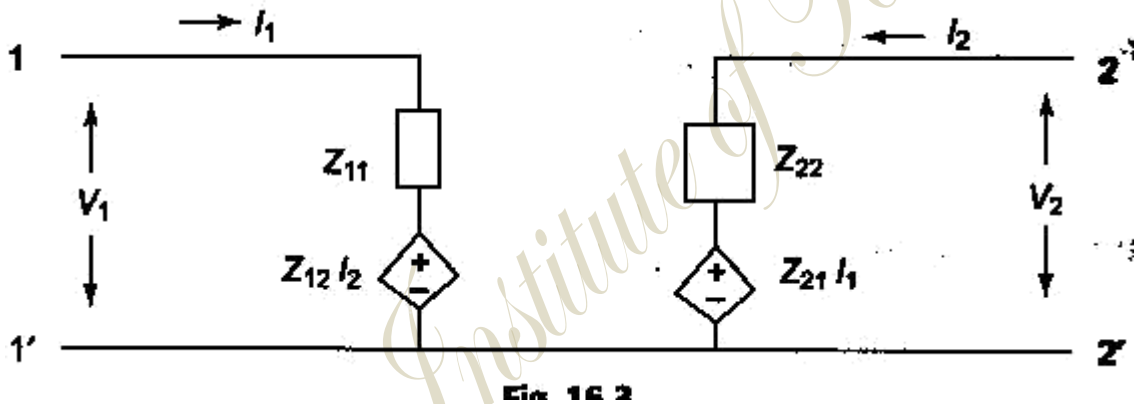


Figure 1.3

If the network under study is reciprocal or bilateral, then in accordance with the reciprocity principle

$$\left.\frac{V_1}{I_2}\right| I_2 = 0 \quad = \left.\frac{V_2}{I_1}\right| I_1 = 0$$

or

$$Z_{21} = Z_{12}$$

It is observed that all the parameters have the dimensions of impedance. Moreover, individual parameters are specified only when the current in one of the ports is zero. This corresponds to one of the ports being open circuited from which the Z parameters also derive the name open circuit impedance parameters.

Problem 1.1

Find the Z parameters for the circuit shown in Figure 1.4

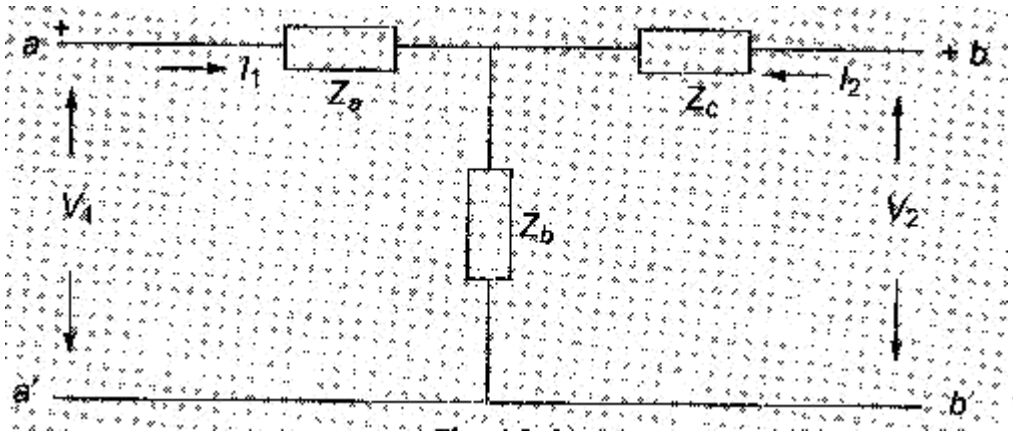


Figure 1.4

Solution The circuit in the problem is a T network. From Eqs 16.1 and 16.2 we have

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{and} \quad V_2 = Z_{21} I_1 + Z_{22} I_2$$

When port b-b' is open circuited,

$$Z_{11} = \frac{V_1}{I_1}$$

Where $V_1 = I_1(z_a + z_b)$

$$\therefore \quad Z_{11} = (Z_a + z_b)$$

$$Z_{21} = \frac{V_2}{I_2}\Big| I_2 = 0$$

Where $V_2 = I_1 Z_b \qquad \therefore Z_{21} = Z_b$

When port a-a' is open circuited, I_1 =0

$$Z_{22} = \frac{V_2}{I_2}\Big| I_1 = 0$$

where $V_2 = I_2(Z_b + Z_c)$

$$Z_{22} = (Z_b + Z_c)$$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1} = 0$$

where $V_1 = I_2 Z_b$ and $Z_{12} = Z_b$

It can be observed that $Z_{12} = Z_{21}$, so the network is a bilateral network which satisfies the principle of reciprocity.

SHORT-CIRCUIT ADMITTANCE (Y) PARAMETERS

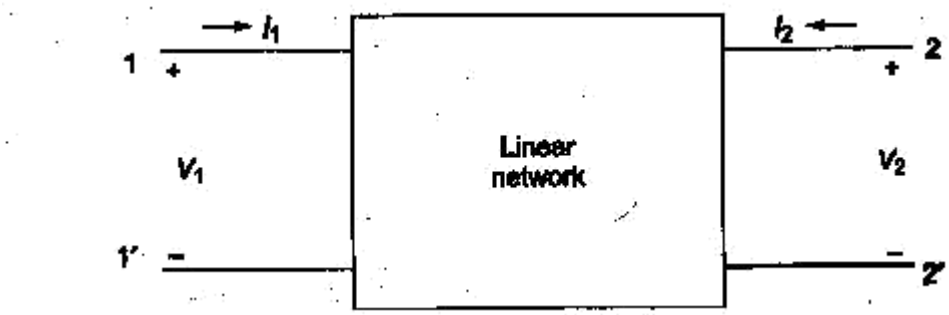


Figure 1.5

A general two- port network which is considered in Section 16.2 is shown in Fig 16.5The Y parameters of a two- port for the positive directions of voltages and currents may be defined by expressing the port currents I_1 and I_2 in terms of the voltages V_1 and V_2. Here V_1, V_2 are dependent variables and V_1 and V_2 are independent variables. I_1 may be considered to be the superposition of two components, one caused by V_1 and the other by V_2.

Thus,

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{..... 1.3}$$

Similarly, ...1.4

Y_{11}, Y_{12}, Y_{21} and Y_{22} are the network network functions and are also called the admittance (Y) parameters. They are defined by Eqs 16.3 and 16.4. These parameters can be represented by matrices as follows

$$[I]=[Y][V]$$

where $I = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$ $Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$ and $V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

Thus ,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

The individual Y parameters for a given network can be defined by setting each port voltage to zero. If we let V_2 be zero by short circuiting port 2-2' then

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

Y_{11} is the driving point admittance at port 1-1', with port 2-2' short circuited.It is also called the short circuit input admittance.

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

Y_{21} is the transfer admittance at port 1-1', with port 2-2' short circuited.It is also called the short

circuited forward transfer admittance. If we let V_1 *be* zero by short circuiting port 1-1',then

$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$

Y_{12} is the transfer admittance at port 2-2', with port 1-1' short circuited. It is also called the short circuited reverse transfer admittance.

$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$

Y_{22} is the short circuit driving point admittance at port 2-2', with port 1-1' short circuited. It is also called the short circuited output admittance.The equivalent circuit of the network governed by equation 1.3 & 1.4 is shown in figure 1.6.

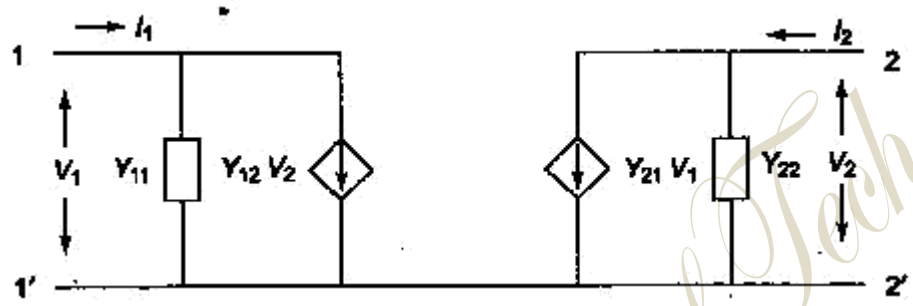


Figure 1.6

If the network under study is reciprocal or bilateral, then in accordance with the reciprocity principle

$\left.\frac{I_2}{V_1}\right|_{V_2=0} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$

or

$Y_{12} = Y_{21}$

It is observed that all the parameters have the dimensions of admittance. Moreover, individual parameters are specified only when the voltage in one of the ports is zero. This corresponds to one of the ports being short circuited from which the Y parameters also derive the name short circuit admittance parameters.

Problem 1.2 Find the Y-parameters for the network shown in Fig.1.7

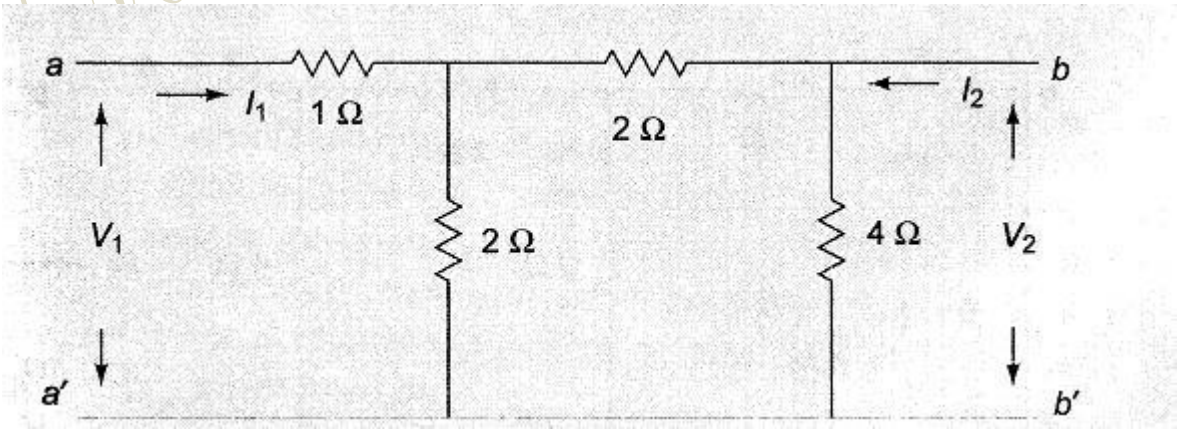


Fig1.7

Solution :

$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$

When b-b' is short circuited, $V_2 = 0$ and the network looks as shown in Fig. 1.8(a)

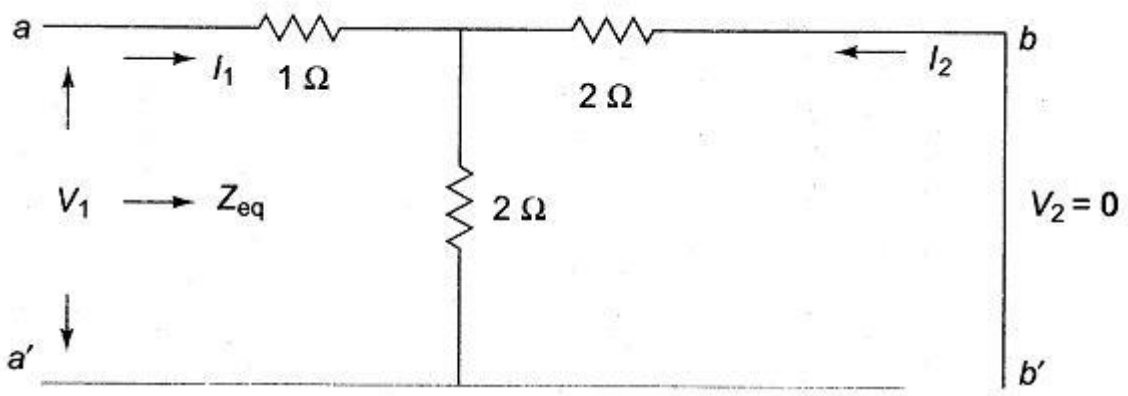


Fig.1.8(a)

$V_1 = I_1 Z_{eq}$

$Z_{eq} = 2\ \Omega$

So, $V_1 = I_1 2$

$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{I_1}{2I_1} = \frac{1}{2}$

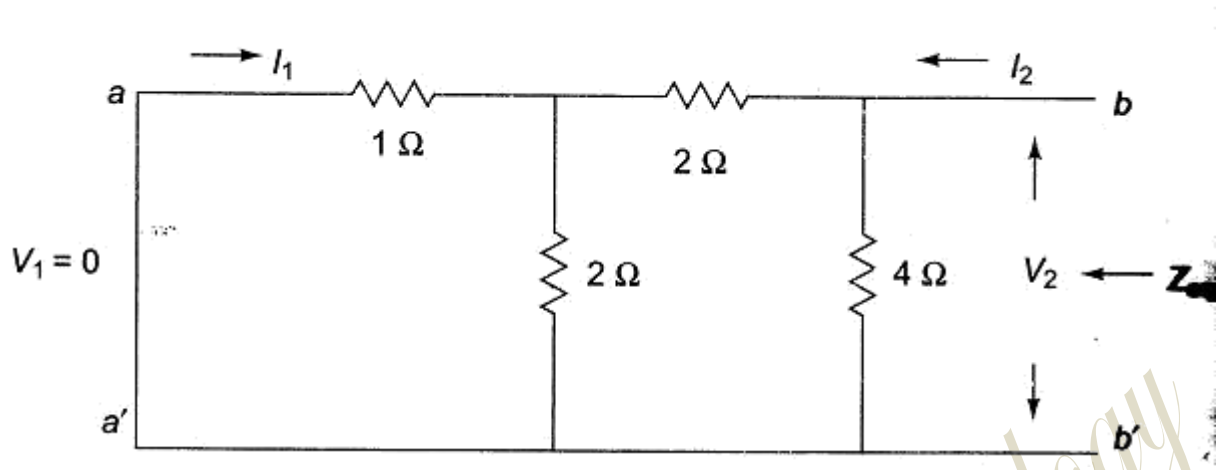
$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$

When b-b' is short circuited, $-I_2 = I_1 \times \frac{2}{4} = \frac{I_1}{2}$

so, $-I_2 = \frac{V_1}{4}$

and $Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\frac{1}{4}$

similarly, when port a-a' is short circuited, $V_1 = 0$ and the network looks as shown in Fig. 1.8(b)



$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$

$V_{2'} = I_2 Z_{eq}$ where Z_{eq} is the equivalent impedance as viewed from b-b′.

$Z_{eq} = \frac{8}{5}\Omega$

$V_{2'} = I_2 \times \frac{8}{5}$

$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{5}{8}$

$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$

with a-a′ is short circuited , $-I_1 = \frac{2}{5} I_2$

Since , $I_2 = 5\,\frac{V_2}{8}$

$-I_1 = \frac{2}{5} \times 5 \frac{V_2}{8} = \frac{V_2}{4}$

So, $Y_{12} = \frac{I_1}{V_2} = -\frac{1}{4}$

The describing equations in terms of tye admittance parameters are

$$I_1 = \frac{1}{2}V_1 + \frac{1}{4}V_2$$

$$I_2 = -\frac{1}{4}V_1 + \frac{5}{8}V_2$$

Transmission (ABCD) parameters

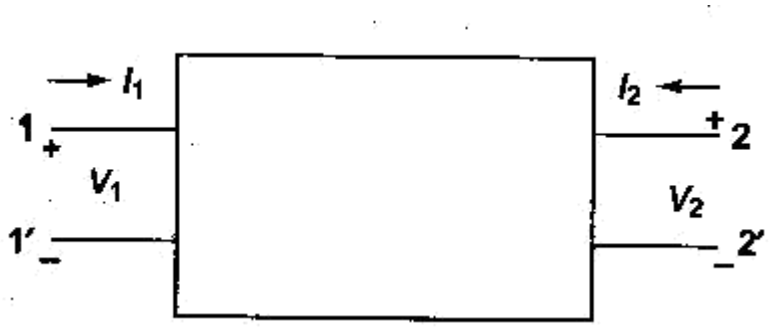


Figure 1.9

Transmission parameters or ABCD parameters are widely used in transmission line theory and cascaded networks. In describing the transmission parameters, the input variables V_1 and I_1 at port 1-1', usually called the sending end are expressed in terms of the output variables V_2 and I_2 at port 2-2', called, the receiving end.The transmission parameters provide a direct relationship between input and output.Transmission patameters are also called general circuit parameters, or chain nparameters. They are defined by

$V_1 = AV_2 - BI_2$ 1.5

$I_1 = CV_2 - DI_2$ 1.6

The negative sign is used with I_2, and not for the parameter B and D. Both the port currents I_1 and I_2 are directed to the right, i.e. with a negative sign in equation a and b the currents at port 2-2' which leaves the port is designated as positive.The parameters A,B,C and d are called Transmission parameters. In the matrix form, equation a and b are expressed as ,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

The matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is called Transmission Matrix.

For a given network, these parameters can be determined as follows. With port 2-2' open circuited i.e. I_2 =0 ; applying a voltage V_1 at the port 1-1', using equ a , we have

A = $\left.\frac{V_1}{V_2}\right| I_2 = 0$ and C = $\left.\frac{I_1}{V_2}\right| I_2 = 0$

hence, $\frac{1}{A} = \left.\frac{V_2}{V_1}\right| I_2 = 0$ $\left.\right| i_2$ =0

1/A is called the open circuit voltage gain a dimension less parameter. And $\frac{1}{C}$ $\left.\frac{V_2}{I_1}\right| i_2 = 0$ $\left.\right| i_2$ =0 is called open circuit transfer impedance. with port 2-2' short circuited, i.e. V_2 =0 , applying voltage V_1 at port 1-1' from equn . b we have

-B = $\left.\frac{V_1}{I_2}\right| v_2 = 0$ and -D = $\left.\frac{I_1}{I_2}\right| v_2 = 0$

$-\frac{1}{B}\left.\frac{I_2}{V_1}\right|_{V_2=0}$ $Y_{21}|_{i_2}$ =0 is called short circuit transfer admittance

and ,

$-\frac{1}{D}\left.\frac{I_2}{I_1}\right|_{V_2=0}$ $\alpha_{21}|_{V_2}$ =0 is called short circuit current gain a dimension less parameter.

Problem 1.3

Find the transmission or general circuit parameters for the circuit shown in Fig.1.10

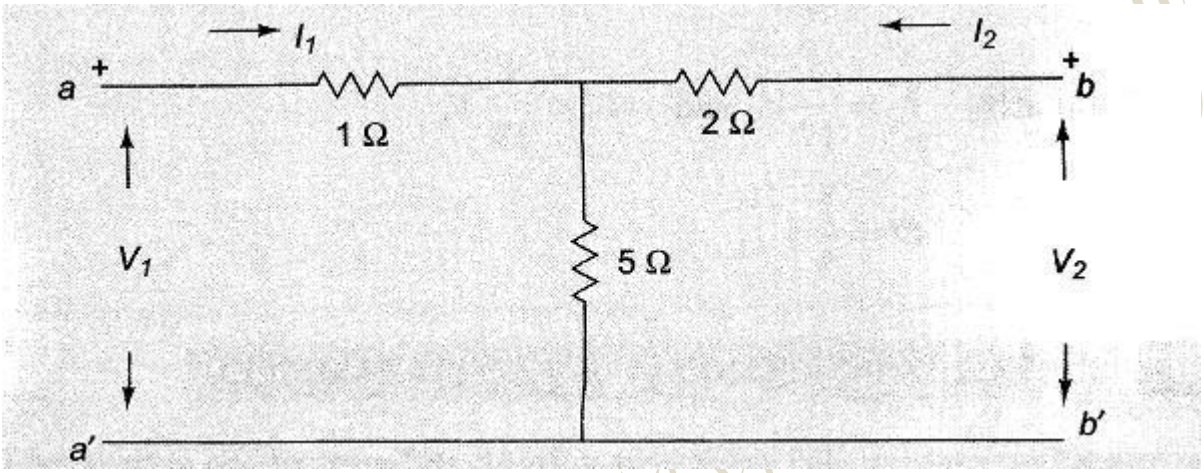


Fig. 1.10

Solution : From Equations 1.5 and 1.6 , we have

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

when b-b' is open circuited i.e. I_2 =0, we have

A = $\left.\frac{V_1}{V_2}\right|_{I_2=0}$

where $V_1 = 6I_1$ and $V_2 = 5I_1$ and hence, A= $\frac{6}{5}$ *and*

C = $\left.\frac{I_1}{V_2}\right|_{I_2=0}$ $=\frac{1}{5}$

when b-b' is short circuited i.e. V_2 =0, we have

B = $-\left.\frac{V_1}{I_2}\right|_{V_2=0}$ and D =- $\left.\frac{I_1}{I_2}\right|_{V_2=0}$

In the circuit, $-I_2 = \frac{5}{17}V_1$ and so, B= $\frac{17}{5}\Omega$

similarly, $I_2 = \frac{7}{17} V_2$ and $-I_2 = \frac{5}{17} V_2$

and hence D = $\frac{7}{5}$

Hybrid parameters

Hybrid parameters or h-parameters find extensive use in transistor circuits. They are well suited to transistor circuits as these parameters can be most conveniently measured. The hybrid matrices describe a two-port network, when the voltage of one port and the current of other port are taken as the independent variables. Consider the network in figure 1.11.

If the voltage at port 1-1' and current at port 2-2' are taken as dependent variables,we can express them in terms of I_1 and V_2 .

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{..... 1.7}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{.....1.8}$$

The coefficient in the above terms are called hybrid parameters.In matrix notation

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

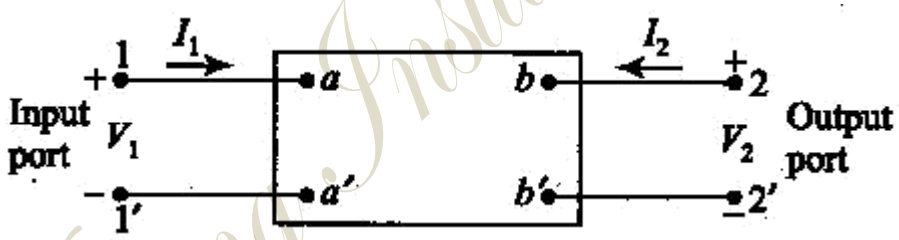


Figure 1.11

from equation a and b the individual h parameters may be defined by letting $I_1 = 0$ and $V_2 = 0$.

when $V_2 = 0$,the port 2-2' is short circuited.

Then $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$ =0 = short circuit input impedance.

$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$ =0 = short circuit forward current gain

Similarly, by letting port 1-1' open, $I_1 = 0$

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$ =0 = open circuit reverse voltage gain

$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$ =0 = open circuited output admittance

Since h-parameters represent dimensionally an impedance, an admittance,a voltage gain and a current gain, they are called hybrid parameters .An equivalent circuit of a two-port network in terms of hybrid parameters is shown below.

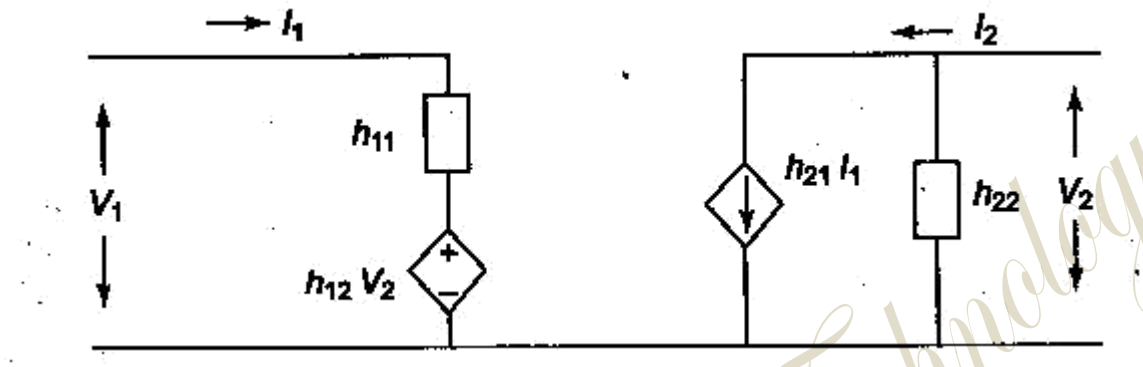


Figure 1.12

Problem 1.4

Find the h-parameters of the network shown in Fig 1.13.

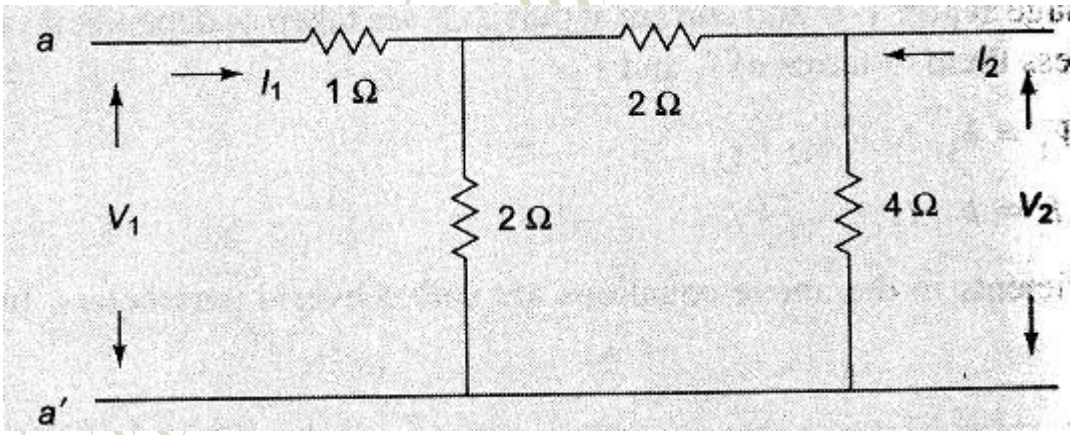


Fig.1.13

Solution :

From equations 1.7 and 1.8 , we have

$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$; $h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$; $h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$; $h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$

If port b-b' is short circuited, V_2 = 0 and the network looks as shown in Fig. 1.14(a)

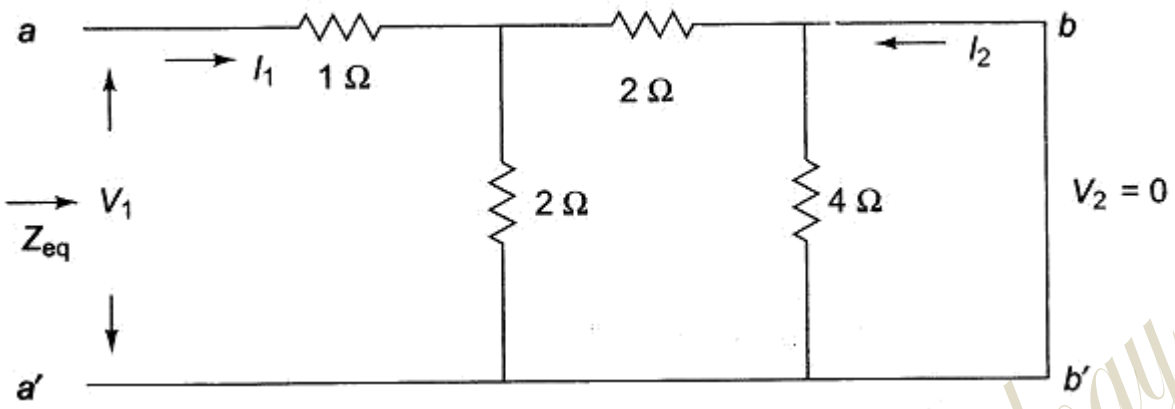


Fig.1.14(a)

$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$; $V_1 = I_1 Z_{eq}$

Z_{eq} is the equivalent impedance as viewed from port a-a′ is 2Ω

so, $V_1 = I_1$ 2 V

$h_{11} = \frac{V_1}{I_1} =$ 2Ω

$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$ =0 when V_2=0; - $=I_2$ $\frac{I_1}{2}$ and hence $h_{21} = -\frac{1}{2}$

If port a-a′ is open circuited, I_1 = 0 and the network looks as shown in Fig. 1.14(b) then

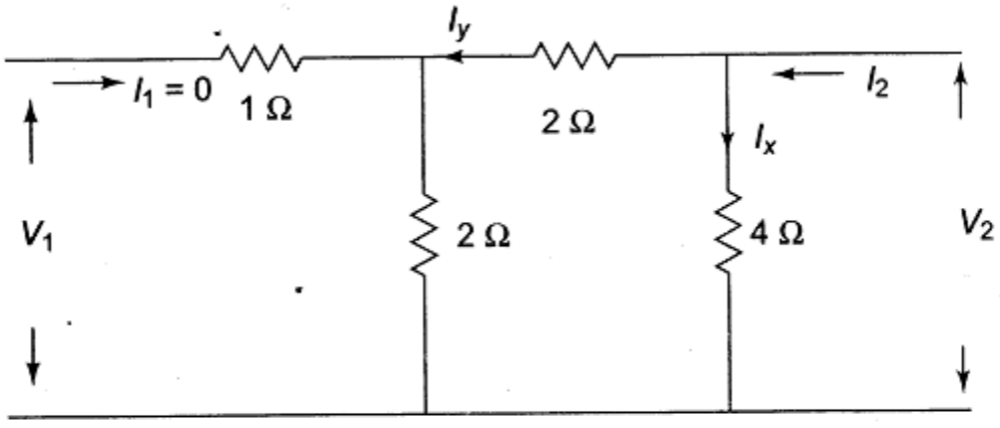


Fig.1.14(b)

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$ and $V_1 = I_x 2$; $I_x = \frac{I_y}{2}$

$V_2 = I_y 4$; $I_y = \frac{I_2}{2}$

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \_$ and $h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \_$

INTER RELATIONSHIPS OF DIFFERENT PARAMETERS

Expression of z parameters in terms of Y parameters and vice-versa

From equations 1.1,1.2,1.3 & 1.4 , it is easy to derive the relation between the open circuit impedance parameters and the short circuit admittance parameters by means of two matrix equations of the respective parameters. By solving equation a and b for I_1 and I_2 , we get

$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$; and $\begin{bmatrix} Z_{11} & Y_1 \\ Z_{21} & V_2 \end{bmatrix}$

where Δ_z is the determinant of Z matrix

$\Delta_z =$

$\frac{Z_{22}}{\Delta_z}$ 1.9

$\frac{Z_{12}}{\Delta_z}$

$\frac{Z_{21}}{\Delta_z}$ 1.10

$\frac{Z_{11}}{\Delta_z}$

comparing equations 1.9 and 1.10 with equations 1.3 and 1.4 we have

$Y_{11} = \frac{Z_{22}}{\Delta_z}$; $Y_{12} = -\frac{Z_{12}}{\Delta_z}$

$Y_{21} = -\frac{Z_{21}}{\Delta_z}$; $Y_{22} = \frac{Z_{11}}{\Delta_z}$

In a similar manner, the z parameters may be expressed in terms of the admittance parameters by solving equations 1.3 and 1.4 for V_1 and V_2.

$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$; and $\begin{bmatrix} Y_{11} & I_1 \\ Y_{21} & I_2 \end{bmatrix}$

where Δ_y is the determinant of Y matrix

$\Delta_y =$

$\frac{Y_{22}}{\Delta_y}$ $\frac{Y_{12}}{\Delta_y}$

……1.11

$\frac{Y_{22}}{\Delta y}$ $\frac{Y_{22}}{\Delta y}$ …….. 1.12

comparing equations 1.11 and 1.12 with equations 1.1 and 1.2 we have

$Z_{11} = \frac{Y_{22}}{\Delta_y}$; $Z_{12} = -\frac{Y_{12}}{\Delta_y}$

$Z_{21} = -\frac{Y_{21}}{\Delta_y}$; $Z_{22} = \frac{Y_{11}}{\Delta_y}$

General Circuit Parameters or ABCD Parameters in Terms of Z parameters and Y Parameters

We know that

$V_1 = AV_2 - BI_2$; ; $I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_1 = CV_2 - DI_2$; ; $I_2 = Y_{21}V_1 + Y_{22}V_2$

A= $\left.\frac{V_1}{V_2}\right|_{I_2=0}$; C= $\left.\frac{I_1}{V_2}\right|$; B=- $\left.\frac{V_1}{I_2}\right|_{V_2=0}$; D=- $\left.\frac{I_1}{I_2}\right|_{V_2=0}$

Substituting the condition I_2 =0 in equations 1.1 and 1.2 we get

A = $\left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{Z_{11}}{Z_{21}}$

Substituting the condition I_2 =0 in equations 1.4 we get ,

A = $\left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{Y_{22}}{Y_{12}}$

Substituting the condition I_2 =0 in equations 1.2 we get

C = $\left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{Z_{21}}$

Substituting the condition I_2 =0 in equation 1.3 and 1.4 and solving for V_2 gives $-I_1\frac{Y_{12}}{\Delta_y}$

Where Δ_y is the determinant of the admittance matrix

$\left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{-\Delta_y}{Y_{21}}$ = C

Substituting the condition V_2 =0 in equations 1.4, we get

$\left.\frac{V_1}{I_2}\right|_{V_2=0} = -\frac{1}{Y_{21}}$ = B

Substituting the condition V_2 =0 in equation 1.1 and 1.2 and solving for I_2 gives $-V_1\frac{Z_{12}}{\Delta_z}$

Where Δ_{ss} is the determinant of the impedance matrix

$$-\left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{\Delta_Z}{Z_{21}} = B$$

Substituting the condition $V_{2}=0$ in equation 1.2 we get ,

$$\left.\frac{-I_1}{I_2}\right|_{V_2=0} = \frac{Z_{22}}{Z_{21}} = D$$

Substituting the condition $V_{2}=0$ in equations 1.3 and 1.4

we get

$$\left.\frac{-I_1}{I_2}\right|_{V_2=0} = \frac{-Y_{11}}{Y_{21}} = D$$

T and π representation

A two-port network with any number of elements may be converted into a two-port three-element network. Thus, a two-port network may be represented by an equivalent T-network, i.e. three impedances are connected together in the form of a T as shown in figure 1.15.

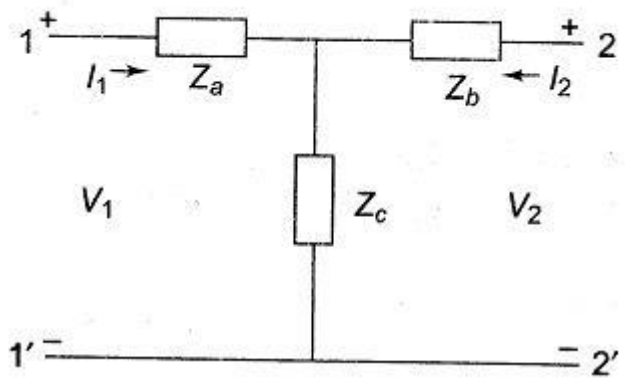


Figure 1.15

It is possible to express the elements of the T-network in term of Z parameters,or ABCD parameters as explained below.

Z parameters of the network

$$Z_{11} \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_a + Z_c$$

$$Z_{21} \left.\frac{V_2}{I_1}\right|_{I_2=0} = Z_c$$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1 = 0} = Z_b + Z_c$$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1 = 0} = Z_c$$

From the above relations, it is clear that

$$Z_a = Z_{11} - Z_{21}$$

$$Z_b = Z_{22} - Z_{12}$$

$$Z_c = Z_{12} - Z_{21}$$

ABCD parameters of the network

$$A = \frac{V_1}{V_2}\Big|_{I_2 = 0} = \frac{Z_a + Z_c}{Z_c}$$

$$B = \frac{-V_1}{I_2}\Big|_{V_2 = 0}$$

When 2- $2'$ is short circuited

$$-I_2 = \frac{V_1 Z_c}{Z_b Z_c + Z_a (Z_b + Z_c)}$$

$$B = (Z_a + Z_b) + \frac{Z_a Z_b}{Z_c}$$

$$C = \frac{I_1}{V_2}\Big|_{I_2 = 0} = \frac{1}{Z_c}$$

$$D = \frac{-I_1}{I_2}\Big|_{V_2 = 0}$$

When 2- $2'$ is short circuited

$$-I_2 = I_1 \frac{Z_c}{Z_b + Z_a}$$

$$D = \frac{Z_b + Z_c}{Z_c}$$

From the above relations we can obtain

$$Z_a = \frac{A-1}{C} \; ; Z_b = \frac{D-1}{C} \; Z_c = \frac{1}{C}$$

Problem :1.6

The Z parameters of a Two-port network are $Z_{11} = 10\Omega$, $Z_{12} = 15\Omega$, $Z_{12} = Z_{21} = 5\Omega$.

Find the equivalent T network and ABCD Parameters.

Solution :

The equivalent T network is shown in Figure 1.16

where $Z_a = Z_{11} - Z_{21} = 5\Omega$

$Z_b = Z_{22} - Z_{12} = 10\Omega$

and $Z_c = 5\Omega$

The ABCD parameters of the network are

$A = \frac{Z_a}{Z_c} + 1 = 2$; $B = (Z_a + Z_b) + \frac{Z_a Z_b}{Z_c} = 25\Omega$

$C = \frac{1}{Z_c} = 0.02$; $D = 1 + \frac{Z_b}{Z_c} = 3$

In a similar way a two-port network may be represented by an equivalent π - network, i.e. three impedances or admittances are connected together in the form of π as shown in Fig 1.17.

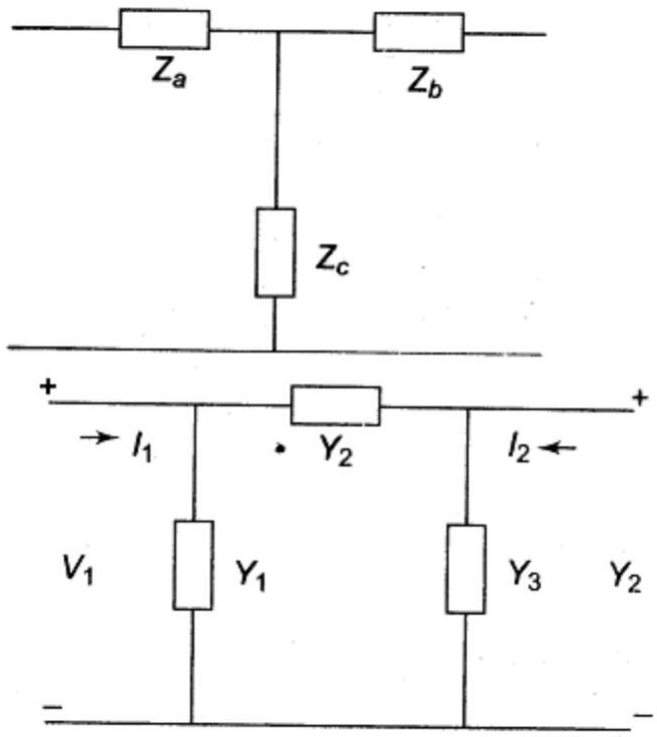


Fig. 1.16 Fig.1.17

It is possible to express the elements of the π -network in terms of Y parameters or ABCD parameters as explained below.

Y-parameters of the network

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = Y_1 + Y_2$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -Y_2$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = Y_3 + Y_2$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -Y_2$$

Writing ABCD parameters in terms of Y parameters yields the following results.

$$A = \frac{-Y_{22}}{Y_{21}} = \frac{Y_3 + Y_2}{Y_2}$$

$$B = \frac{-1}{Y_{21}} = \frac{1}{Y_2}$$

$$C = \frac{-\Delta Y}{Y_{21}} = Y_1 + Y_2 + \frac{Y_1 Y_3}{Y_2}$$

$$D = \frac{-Y_{11}}{Y_{21}} = \frac{Y_1 + Y_2}{Y_2}$$

from the above results, we obtain

$$Y_1 = \frac{D-1}{B}\ ;\ Y_3 = \frac{A-1}{B}\ ;$$

$$Y_2 = \frac{1}{B}$$

……XXXXXXXXXXXXXXXXXX…….

9.1 CLASSIFICATION OF FILTERS

A filter is a reactive network that freely passes the desired band of frequencies while almost totally suppressing all other bands. A filter is constructed from purely reactive elements, for otherwise the attenuation would never becomes zero i n the pass band of the filter network. Filters differ from simple resonant circuit in providing a substantially constant transmission over the band which they accept; this band may lie between any limits depending on the design. Ideally, filters should produce no attenuation in the desired band, called the transmission band or pass band, and should provide total or infinite attenuation at all other frequencies, called attenuation band or stop band. The frequency which separates the transmission band and the attenuation band is defined as the cut-off frequency of the wave filters, and is designated by *fc*

Filter networks are widely used in communication systems to separate various voice channels in carrier frequency telephone circuits. Filters also find applications in instrumentation, telemetering equipment etc. where it is necessary to transmit or attenuate a limited range of frequencies. A filter may, in principle, have any number of pass bands separated by attenuation bands.However, they are classified into four common types, viz.low pass, high pass, band pass and band elimination.

Decibel and neper

The attenuation of a wave filter can be expressed in decibels or nepers.Neper is defined as the natural logarithm of the ratio of input voltage (or current) to the output voltage (or current), provide that the network is properly terminated in its characteristic impedance Z_0 .

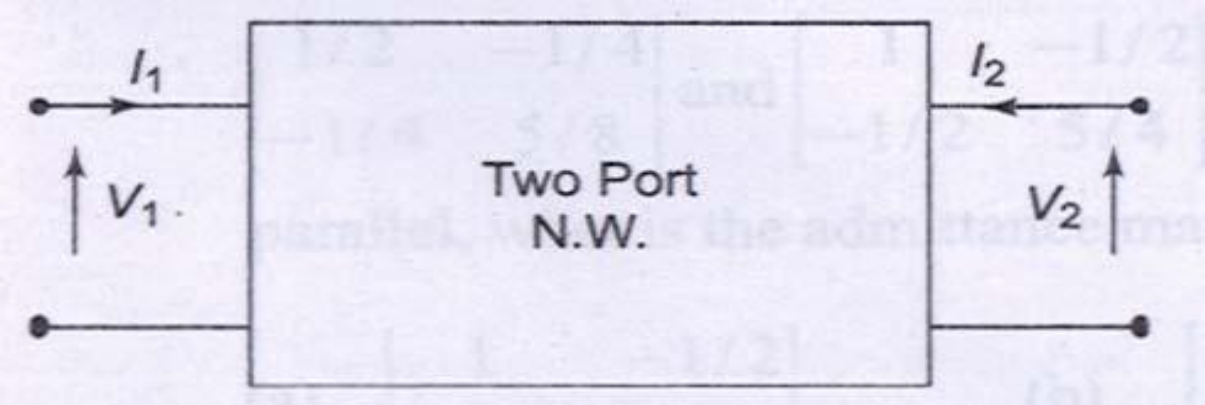


Fig .9.1 (a)

From fig. 9.1 (a) the number of nepers, N= *log* e $[V_1/V_2]$ or *log*e $[I_1/I_2]$. A neper can also be expressed in terms of input power,P_1 and the output power P_2 as $N=1/2 \log_e P_1/P_2$. A decibel is defined as ten times the common logarithms of the ratio of the input power to the output power.

Decibel $D=10 \log_{10} P_1/P_2$

The decibel can be expressed in terms of the ratio of input voltage (or current) and the output voltage (or current.)

$D=20 \log_{10}[V_1/V_2] = 20 \log_{10}[I_1/I_2]$

* One decibel is equal to 0.115 N.

Low Pass Filter

By definition a low pass (LP) filter is one which passes without attenuation all frequencies up to the cut-off frequency f_c , and attenuates all other frequencies greater than f_c .The attenuation characteristic of an ideal LP filter is shown in fig.9.1(b).This transmits currents of all frequencies from zero up to the cut-off frequency. The band is called pass band or transmission band.Thus,the pass band for the LP filter is the frequency range 0 to f_c.The frequency range over which transmission does not take place is called the stop band or attenuation band. The stop band for a LP filter is the frequency range above f_c .

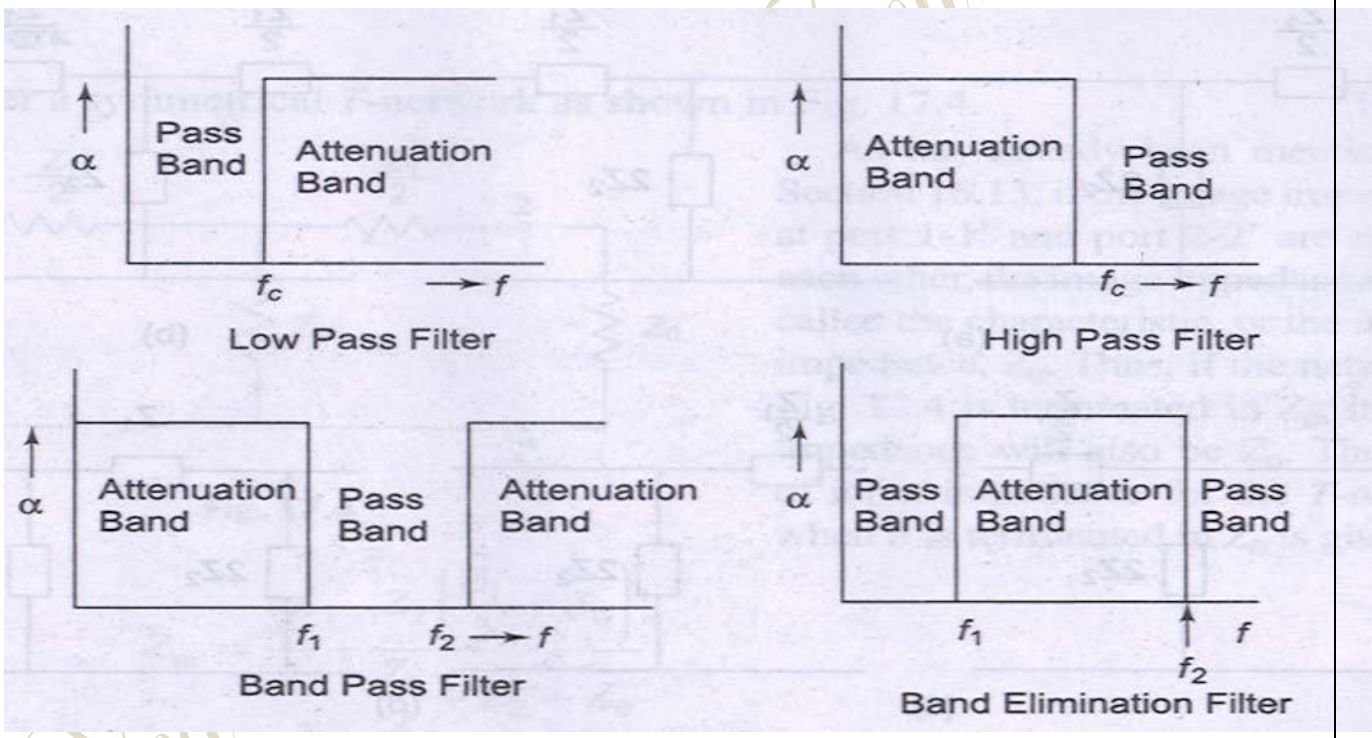


Fig.9.1 (b)

High Pass Filter

A high pass (HP) filter attenuates all frequencies below a designated cut-off frequency, f_c , and passes all frequencies above f_c . Thus the pass band of this filter is the frequency range above f_c, and the stop band is the frequency range below f_c . The attenuation characteristic of a HP filter is shown in fig.9.1 (b).

Band Pass Filter

A band pass filter passes frequencies between two designated cut-off frequencies and attenuates all other frequencies. It is abbreviated as BP filter. As shown in fig.9.1 (b), a BP filter has two cut-off frequencies and will have the pass band $f_2 - f_1$; f_1 is called the lower cut –off frequency, while f_2 is called the upper cut-off frequency.

Band Elimination filter

A band elimination filter passes all frequencies lying outside a certain range, while it attenuates all frequencies between the two designated frequencies. It is also referred as band stop filter. The characteristic of an ideal band elimination filter is shown in fig.9.1 (b). All frequencies between f_1 and f_2 will be attenuated while frequencies below f_1 and above f_2 will be passed.

9.2 FILTER NETWORKS

Ideally a filter should have zero attenuation in the pass band. This condition can only be satisfied if the elements of the filter are dissipationless.which cannot be realized in practice. Filters are designed with an assumption that the elements of the filters are purely reactive. Filters are made of symmetrical T, or π section. T and π section can be considered as combination of unsymmetrical L sections as shown in Fig.9.2.

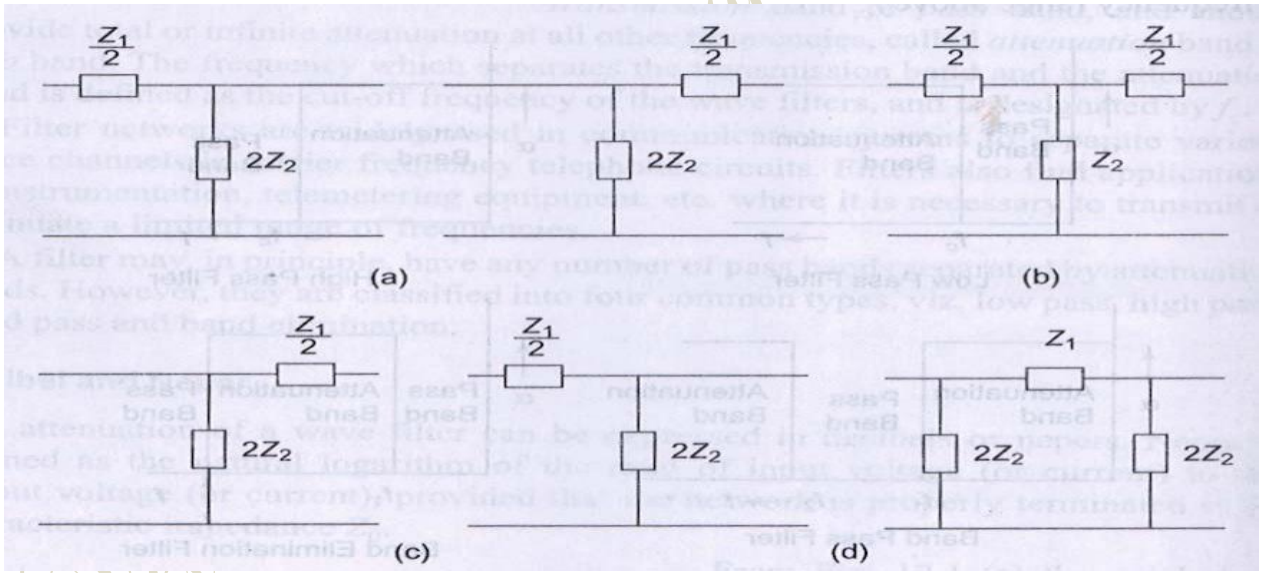


Fig. 9.2

The ladder structure is one of the commonest forms of filter network. A cascade connection of several T and π sections constitutes a ladder network. A common form of the ladder network is shown in Fig.9.3.

Figure 9.3(a) represents a T section ladder network, whereas Fig.9.3 (b) represents the π section ladder network. It can be observed that both networks are identical except at the ends.

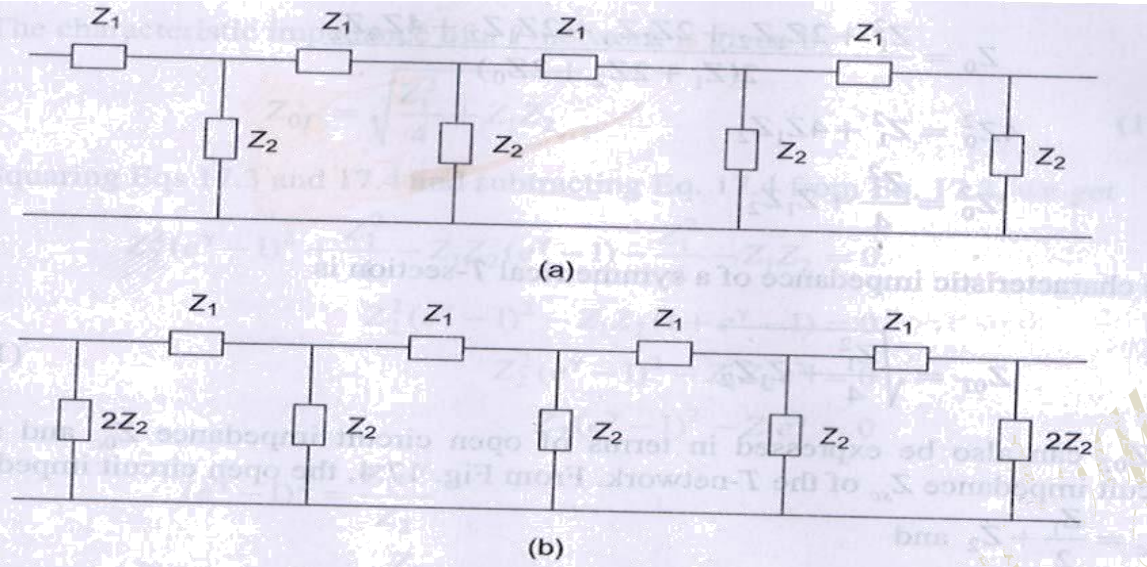


Fig. 9.3

9.3 EQUATIONS OF FILTER NETWORKS

The study of the behavior of any filter requires the calculation of its propagation constant ɣ, attenuation α, phase shift β and its characteristic impedance Z_0.

T-Network

Consider a symmetrical T-network as shown in Fig. 9.4.

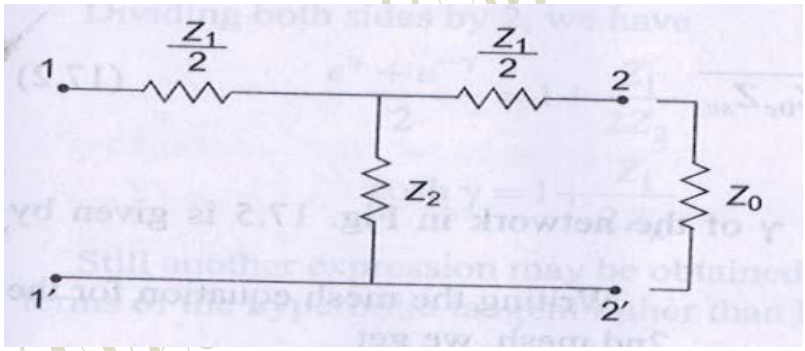


Fig.9.4

If the image impedances at port 1-1' and port 2-2' are equal to each other ,the image impedance is then called the characteristic, or the iterative impedance, Z_0 .Thus, if the network in Fig.9.4 is terminated in Z_0, its input impedance will also be Z_0. The value of input impedance for the T-network when it is terminated in Z_0 is given by

$$Z_{in} = \frac{Z_1}{2} + \frac{Z_2\left(\frac{Z_1}{2} + Z_0\right)}{\frac{Z_1}{2} + Z_2 + Z_0}$$

also $Z_{in} = Z_0$

$$\therefore \quad Z_0 = \frac{Z_1}{2} + \frac{2Z_2\left(\frac{Z_1}{2} + Z_0\right)}{Z_1 + 2Z_2 + 2Z_0}$$

$$Z_0 = \frac{Z_1}{2} + \frac{(Z_1Z_2 + 2Z_2Z_0)}{Z_1 + 2Z_2 + 2Z_0}$$

$$Z_0 = \frac{Z_1^2 + 2Z_1Z_2 + 2Z_1Z_0 + 2Z_1Z_2 + 4Z_0Z_2}{2(Z_1 + 2Z_2 + 2Z_0)}$$

$$4Z_0^2 = Z_1^2 + 4Z_1Z_2$$

$$Z_0^2 = \frac{Z_1^2}{4} + Z_1Z_2$$

The characteristic impedance of a symmetrical T-section is

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1Z_2}$$

(9.1)

Z_{0T} can also be expressed in terms of open circuit impedance Z_{0C} and short circuit impedance Z_{SC} of the T – network . From Fig. 9.4, the open circuit impedance $Z_{0C} = Z_1/2 + Z_2$ and

$$Z_{sc} = \frac{Z_1}{2} + \frac{\frac{Z_1}{2} \times Z_2}{\frac{Z_1}{2} + Z_2}$$

$$Z_{sc} = \frac{Z_1^2 + 4Z_1Z_2}{2Z_1 + 4Z_2}$$

$$Z_{0c} \times Z_{sc} = Z_1Z_2 + \frac{Z_1^2}{4}$$

$$= Z_{0T}^2 \quad \text{or} \quad Z_{0T} = \sqrt{Z_{0c}Z_{sc}}$$

(9.2)

Propagation Constant of T- Network

By definitation the propagation constant У of the network in Fig.9.5 is given by $У = \log_e I_1/I_2$

Writing the mesh equation for the 2nd mesh, we get

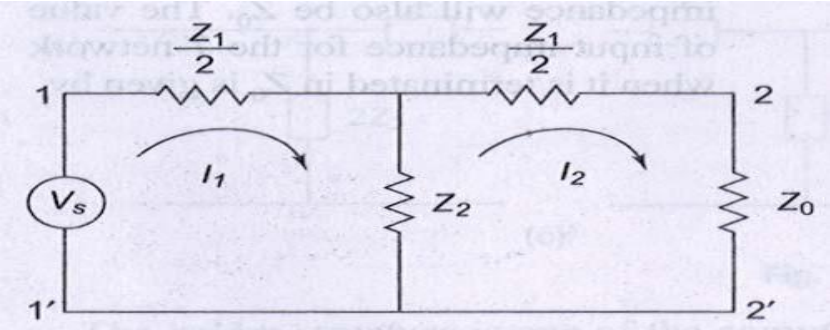


Fig.9.5

$$I_1 Z_2 = I_2\left(\frac{Z_1}{2} + Z_2 + Z_0\right)$$

$$\frac{I_1}{I_2} = \frac{\frac{Z_1}{2} + Z_2 + Z_0}{Z_2} = e^{\gamma}$$

$$\therefore \quad \frac{Z_1}{2} + Z_2 + Z_0 = Z_2 e^{\gamma}$$

$$Z_0 = Z_2(e^{\gamma} - 1) - \frac{Z_1}{2} \qquad (9.3)$$

The characteristic impedance of a T – network is given by

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} \qquad (9.4)$$

Squaring Esq. 9.3 and 9.4 and subtracting Eq.9.4 from Eq.9.3, we get

$$Z_2^2(e^{\gamma}-1)^2+\frac{Z_1^2}{4}-Z_1Z_2(e^{\gamma}-1)-\frac{Z_1^2}{4}-Z_1Z_2=0$$

$$Z_2^2(e^{\gamma}-1)^2-Z_1Z_2(1+e^{\gamma}-1)=0$$

$$Z_2^2(e^{\gamma}-1)^2-Z_1Z_2e^{\gamma}=0$$

$$Z_2(e^{\gamma}-1)^2-Z_1e^{\gamma}=0$$

$$(e^{\gamma}-1)^2=\frac{Z_1e^{\gamma}}{Z_2}$$

$$e^{2\gamma}+1-2e^{\gamma}=\frac{Z_1}{Z_2e^{-\gamma}}$$

Rearranging the above equation, we have

$$e^{-\gamma}(e^{2\gamma}+1-2e^{\gamma})=\frac{Z_1}{Z_2}$$

$$(e^{\gamma}+e^{-\gamma}-2)=\frac{Z_1}{Z_2}$$

Dividing both sides by 2, we have

$$\frac{e^{\gamma}+e^{-\gamma}}{2}=1+\frac{Z_1}{2Z_2}$$

$$\cosh\gamma=1+\frac{Z_1}{2Z_2}$$

(9.5)

Still another expression may obtained for the complex propagation constant in terms of the hyperbolic tangent rather than hyperbolic cosine.

$$\sinh \gamma = \sqrt{\cos h^2 \gamma - 1}$$

$$= \sqrt{\left(1 + \frac{Z_1}{2Z_2}\right)^2 - 1} = \sqrt{\frac{Z_1}{Z_1} + \left(\frac{Z_1}{2Z_2}\right)^2}$$

$$\sinh \gamma = \frac{1}{Z_2}\sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}} = \frac{Z_{0T}}{Z_2}$$

(9.6)

Dividing Eq.9.6 by Eq.9.5, We get

$$\tanh \gamma = \frac{Z_{0T}}{Z_2 + \frac{Z_1}{2}}$$

But $$Z_2 + \frac{Z_1}{2} = Z_{0c}$$

Also from Eq. 9.2,

$$Z_{0T} = \sqrt{Z_{0c} Z_{sc}}$$

$$\tanh \gamma = \sqrt{\frac{Z_{sc}}{Z_{0c}}}$$

Also $$\sinh \frac{\gamma}{2} = \sqrt{\frac{1}{2}(\cosh \gamma - 1)}$$

Where $$\cosh \gamma = 1 + (Z_1 / 2Z_2)$$

$$= \sqrt{\frac{Z_1}{4Z_2}}$$

(9.7)

π – Network

Consider asymmetrical π – section shown in Fig. 9.6. When the network is terminated in Z_0 at port 2 – 2′ its input impedance is given by

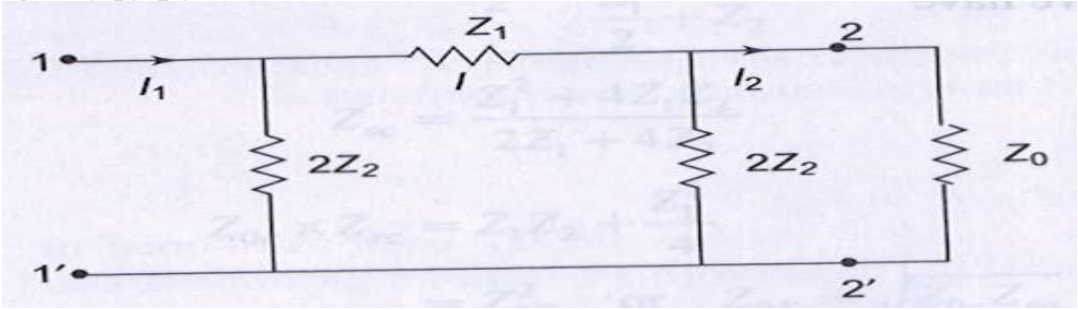


Fig.9.6

$$Z_{in} = \frac{2Z_2\left[Z_1 + \frac{2Z_2 Z_0}{2Z_2 + Z_0}\right]}{Z_1 + \frac{2Z_2 Z_0}{2Z_2 + Z_0} + 2Z_2}$$

By definition of characteristic impedance, $Z_{in} = Z_0$

$$Z_0 = \frac{2Z_2\left[Z_1 + \frac{2Z_2 Z_0}{2Z_2 + Z_0}\right]}{Z_1 + \frac{2Z_2 Z_0}{2Z_2 + Z_0} + 2Z_2}$$

$$Z_0 Z_1 + \frac{2Z_2 Z_0^2}{2Z_2 + Z_0} + 2Z_0 Z_2 = \frac{2Z_2(2Z_1 Z_2 + Z_0 Z_1 + 2Z_0 Z_2)}{(2Z_2 + Z_0)}$$

$$2Z_0 Z_1 Z_2 + Z_1 Z_0^2 + 2Z_0^2 Z_2 + 4Z_2^2 Z_0 + 2Z_2 Z_0^2 = 4Z_1 Z_2^2 + 2Z_0 Z_1 Z_2 + 4Z_0 Z_2^2$$

$$Z_1 Z_0^2 + 4Z_2 Z_0^2 = 4Z_1 Z_2^2$$

$$Z_0^2 (Z_1 + 4Z_2) = 4Z_1 Z_2^2$$

$$Z_0^2 = \frac{4Z_1 Z_2^2}{Z_1 + 4Z_2}$$

Rearranging the above equation leads to

$$Z_0 = \sqrt{\frac{Z_1 Z_2}{1 + Z_1 / 4Z_2}}$$

(9.8)

which is the characteristic impedance of a symmetrical π-network,

$$Z_{0\pi} = \frac{Z_1 Z_2}{\sqrt{Z_1 Z_2 + Z_1^2 / 4}}$$

From Eq. 9.1

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$\therefore \quad Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}}$$

(9.9)

$Z_{0\pi}$ can be expressed in terms of the open circuit impedance Z_{0C} and short circuit impedance Z_{SC} of the π network shown in Fig.9.6 exclusive of the load Z_0.

From Fig.9.6, the input impedance at port 1- 1′ when port 2 – 2′ is open is given by

$$Z_{0C} = \frac{2Z_2(Z_1 + 2Z_2)}{Z_1 + 4Z_2}$$

Similarly, the input impedance at port 1 – 1′ when port 2 – 2′ is short circuit is given by

$$Z_{sc} = \frac{2Z_1Z_2}{2Z_2 + Z_1}$$

$$\text{Hence} \quad Z_{0c} \times Z_{sc} = \frac{4Z_1Z_2^2}{Z_1 + 4Z_2} = \frac{Z_1Z_2}{1 + Z_1/4Z_2}$$

Thus from Eq. 9.8

$$Z_{0\pi} = \sqrt{Z_{0c}\ Z_{sc}}$$

(9.10)

Propagation Constant of π – Network

The propagation constant of a symmetrical π – section is the same as that for a symmetrical T – Section.

$$\text{i.e.} \quad \cosh \gamma = 1 + \frac{Z_1}{2Z_2}$$

9.4 CLASSIFICATION OF PASS BAND AND STOP BAND

It is possible to verify the characteristics of filters from the propagation constant of the network. The propagation constant У, being a function of frequency, the pass band, stop band and the cut-off point, i.e. the point of separation between the two bands, can be identified. For symmetrical T or π – section, the expression for propagation constant У in terms of the hyperbolic functions is given by Eqs 9.5 and 9.7 in section 9.3. From Eq.9.7, sin h У/2 = √(Z_1 /4Z_2) .

If Z_1 and Z_2 are both pure imaginary values, their ratio, and hence Z_1 /4Z_2 , will be a pure real number. Since Z_1 and Z_2 may be anywhere in the range from -jα to +jα , Z_1 / 4Z_2 may also have any

real value between the infinite limits . Then sin h $\gamma/2 = \sqrt{Z_1}/\sqrt{4Z_2}$ will also have infinite limits, but may be either real or imaginary depending upon whether $Z_1 / 4Z_2$ is positive or negative.

We know that the propagation constant is a complex function $\gamma = \alpha+j\beta$, the real part of the complex propagation constant α , is a measure of the change in magnitude of the current or voltage in the network ,known as the attenuation constant . β is a measure of the difference in phase between the input and output currents or voltages. Known as phase shift constant Therefore α and β take on different values depending upon the of $Z_1/ 4Z_2$. From Eq.9.7, We have

$$\sinh\frac{\gamma}{2} = \sinh\left(\frac{\alpha}{2}+\frac{j\beta}{2}\right) = \sinh\frac{\alpha}{2}\cos\frac{\beta}{2} + j\cosh\frac{\alpha}{2}\sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

(9.11)

Case A

If Z_1 and Z_2 are the same type of reactances, then $[Z_1 / 4Z_2]$ is real and equal to say $\alpha+x$.

The imaginary part of the Eq. 9.11 must be zero.

$$\therefore \quad \cosh\frac{\alpha}{2}\sin\frac{\beta}{2} = 0$$

(9.12)

$$\sinh\frac{\alpha}{2}\cos\frac{\beta}{2} = x$$

(9.13)

α and β must satisfy both the above equations.

Equation 9.12 can be satisfied if $\beta/2 = 0$ or $n\pi$, where n = 0, 1, 2,….., then $\cos \beta/2 = 1$ and $\sinh \alpha/2 = x = \sqrt{(Z_1 / 4Z_2)}$

That x should be always positive implies that

$$\left|\frac{Z_1}{4Z_2}\right| > 0 \text{ and } \alpha = 2\sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$$

(9.14)

Since $\alpha \neq 0$, it indicates that the attenuation exists.

Case B

Consider the case of Z_1 and Z_2 being opposite type of reactances, i.e. $Z_1 / 4Z_2$ is negative , making $\sqrt{Z_1 / 4Z_2}$ imaginary and equal to say Jx

*The real part of the Eq.9.11 must be zero.

$$\sinh \frac{\alpha}{2} \cos \frac{\beta}{2} = 0$$

(9.15)

$$\cosh \frac{\alpha}{2} \sin \frac{\beta}{2} = x$$

(9.16)

Both the equations must be satisfied simultaneously by α and β. Equation 9.15 may be satisfied when $\alpha = 0$, or when $\beta = \pi$. These conditions are considered separately hereunder

(i) When $\alpha = 0$; from Eq. 9.15, $\sinh \alpha/2 = 0$.and from Eq.9.16 $\sin \beta/2 = x = \sqrt{(Z_1 / 4Z_2)}$. But the sine can have a maximum value of 1. Therefore, the above solution is valid only for negative $Z_1 / 4Z_2$, and having maximum value of unity. It indicates the condition of pass band with zero attenuation and follows the condition as

$$-1 \leq \frac{Z_1}{4Z_2} \leq 0$$

$$\beta = 2 \sin^{-1} \sqrt{\frac{Z_1}{4Z_2}}$$

(9.17)

(ii) When $\beta = \pi$, from Eq.9.15, $\cos \beta/2 = 0$. And from Eq.9.16, $\sin \beta/2 = \pm 1$; $\cosh \alpha/2 = x = \sqrt{(Z_1 / 4Z_2)}$.

Since $\cosh \alpha/2 \geq 1$, this solution is valid for negative $Z_1 / 4Z_2$,and having magnitude greater than, or equal to unity. It indicates the condition of stop band since $\alpha \neq 0$.

$$-\alpha \leq \frac{Z_1}{4Z_2} \leq -1$$

$$\alpha = 2 \cosh^{-1} \sqrt{\frac{Z_1}{4Z_2}}$$

(9.18)

It can be observed that there are three limits for case A and B. Knowing the values of Z_1 and Z_2 , it is possible to determine the case to be applied to the filter. Z_1 and Z_2 are made of different types of reactances, or combinations of reactances, so that, as the frequency changes, a filter may pass from one case to another. Case A and (ii) in case B are attenuation bands, whereas (i) in case B is the transmission band.

The frequency which separates the attenuation band from pass band or vice versa is called cut-off frequency. The cut-off frequency is denoted by f_C , and is also termed as nominal frequency. Since Z_0 is real in the pass band and imaginary in an attenuation band, f_C is the frequency at which Z_0 changes from being real to being imaginary. These frequencies occur at

$$\frac{Z_1}{4Z_2} = 0 \text{ or } Z_1 = 0 \qquad 9.18(a)$$

$$\frac{Z_1}{4Z_2} = -1 \text{ or } Z_1 + 4Z_2 = 0 \qquad 9.18\ (b)$$

The above conditions can be represented graphically, as in Fig.9.7.

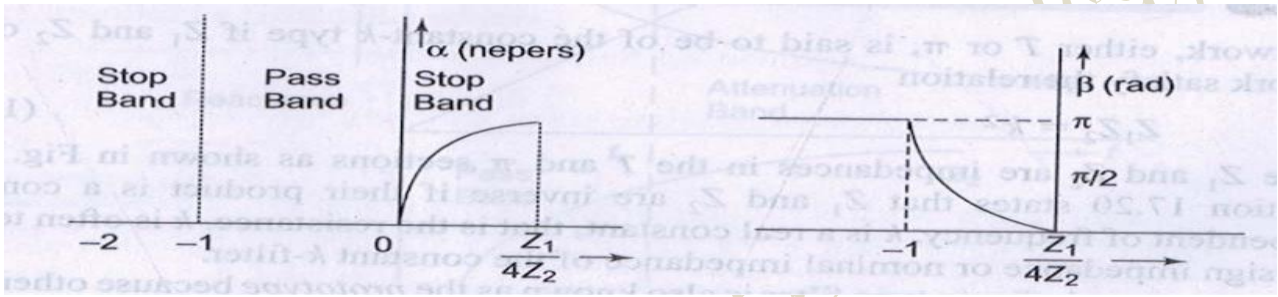


Fig. 9.7

9.5 CHARACTERISTIC IMPEDANCE IN THE PASS AND STOP BANDS

Referring to the characteristic impedance of a symmetrical T-network, from Eq. 9.1 We have

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

If Z_1 and Z_2 are purely reactive, let $Z_1 = jx_1$ and $Z_2 = jx_2$, then

$$Z_{0T} = \sqrt{-x_1 x_2 \left(1 + \frac{x_1}{4x_2}\right)}$$

(9.19)

A pass band exists when x_1 and x_2 are of opposite reactances and

$$-1 < \frac{x_1}{4x_2} < 0$$

Substituting these conditions in Eq. 9.19, we find that Z_{OT} is positive and real. Now consider the stop band. A stop band exists when x_1 and x_2 are of the same type of reactances; then $x_1/4x_2 > 0$. Substituting these conditions in Eq. 9.19, we find that Z_{OT} is purley imaginary in this attenuation region. Another stop band exists when x_1 and x_2 are of the same type of reactances, but with $x_1/4x_2 < -1$.Then from Eq.9.19, Z_{OT} is again purly imaginary in the attenuation region.

Thus, in a pass band if a network is terminated in a pure resistance R_O($Z_{OT} = R_O$), the input impedance is R_O and the network transmits the power received from the source to the R_O without any attenuation. In a stop band Z_{OT} is reactive. Therefore, if the network is terminated in a pure reactance (Z_O = pure reactance), the input impedance is reactive, and cannot receive or transmit power. However, the network transmits voltage and current with 90^0 phase difference and with attenuation. It has already been shown that the characteristics impedance of a symmet rical π- section can be expressed in terms of T. Thus, from Eq.9.9,$Z_{0\pi} = Z_1Z_2/Z_{0T}$.

Since Z_1 and Z_2 are purely reactive, $Z_{0\pi}$ is real, if Z_{OT} is real and Z_{0x} is imaginary if Z_{OT} is imaginary. Thus the conditions developed for T – section are valid for π – sections.

9.6 CONSTANT –K LOW PASS FILTER

A network, either T or π, is said to be of the constant – k type if Z_1 and Z_2 of the network satisfy the relation

$$Z_1Z_2 = k^2$$

(9.20)

Where Z_1 and Z_2 are impedances in the T and π sections as shown in Fig.9.8.Equation 9.20 states that Z_1 and Z_2 are inverse if their product is a constant, independent of frequency. K is a real constant that is the resistance. k is often termed as design impedance or nominal impedance of the constant k – filter.

The constant k, T or π type filter is also known as the *prototype* because other more complex network can be derived from it. A prototype T and π – section are shown in

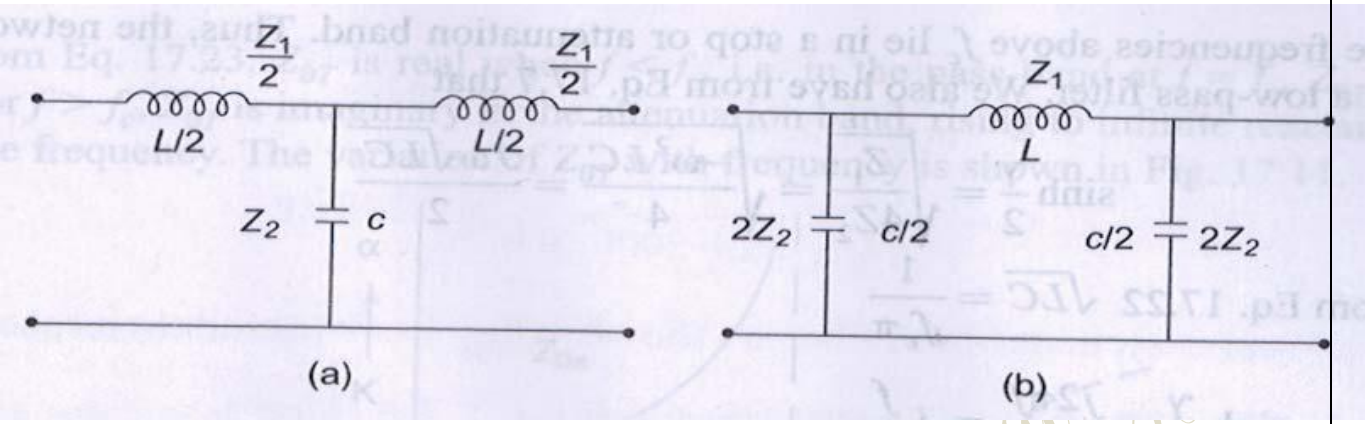


Fig.9.8

Fig.9.8 (a) and (b), where $Z_1 = j\omega_L$ and $Z_2 = 1 / j\omega_C$. Hence $Z_1Z_2 = L/C = k^2$ which is independent of frequency.

$$Z_1Z_2 = k^2 = \frac{L}{C} \quad \text{or} \quad k = \sqrt{\frac{L}{C}}$$

(9.21)

Since the product Z_1 and Z_2 is constant, the filter is a constant – *k* type. From Eq.9.18 (a) the cut-off frequencies are $Z_1 /4Z_2 = 0$,

$$\text{i.e.} \quad \frac{-\omega^2 LC}{4} = 0$$

$$\text{i.e.} \quad f = 0 \text{ and } \frac{Z_1}{4Z_2} = -1$$

$$\frac{-\omega^2 LC}{4} = -1$$

$$\text{or} \quad f_c = \frac{1}{\pi\sqrt{LC}}$$

(9.22)

The pass band can be determined graphically. The reactances of Z_1 and $4Z_2$ will vary with frequency as drawn in Fig.9.9.The cut-off frequency at the intersection of the curves Z_1 and $-4z_2$ is indicated as f_C . On the X – axis as $Z_1 = -4Z_2$ at cut-off frequency, the pass band lies between the frequencies at which $Z_1 = 0$, and $Z_1 = -4Z_2$.

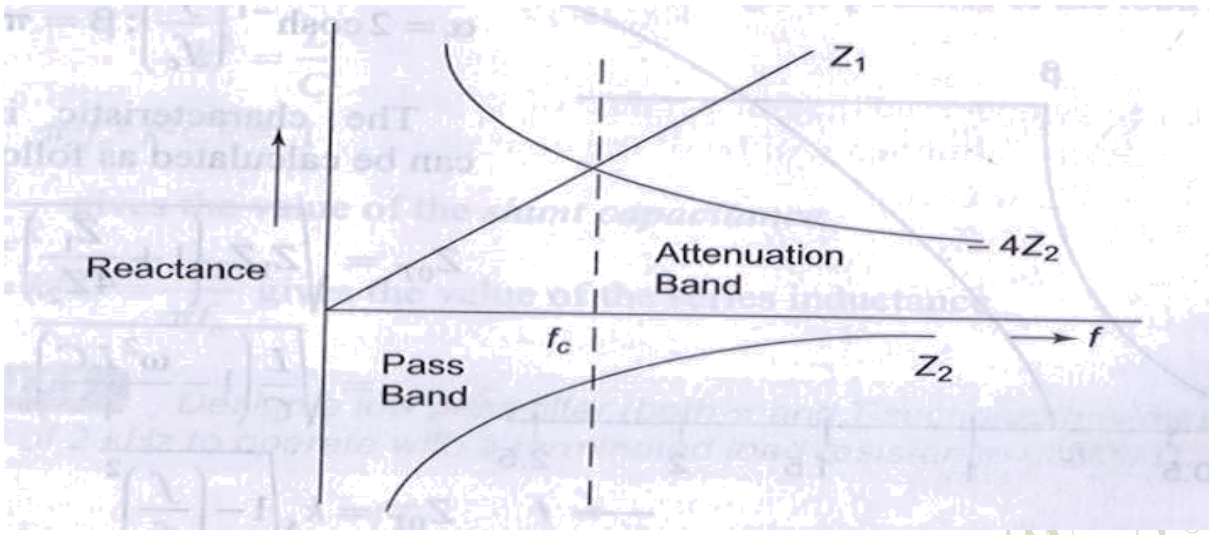


Fig.9.9

All the frequencies above f_C lie in a stop or attenuation band , thus, the network is called a low- pass filter . We also have from Eq.9.7 that

$$\sinh\frac{\gamma}{2}=\sqrt{\frac{Z_1}{4Z_2}}=\sqrt{\frac{-\omega^2LC}{4}}=\frac{J\omega\sqrt{LC}}{2}$$

From Eq.9.22

$$\sqrt{LC}=\frac{1}{f_c\pi}$$

$$\therefore \quad \sinh\frac{\gamma}{2}=\frac{j2\pi f}{2\pi f_c}=j\frac{f}{f_c}$$

We also know that in the pass band

$$-1<\frac{Z_1}{4Z_2}<0$$

$$-1<\frac{-\omega^2LC}{4}<0$$

$$-1<-\left(\frac{f}{f_c}\right)^2<0$$

or $$\frac{f}{f_c}<1$$

and $$\beta=2\sin^{-1}\left(\frac{f}{f_c}\right);\ \alpha=0$$

In the attenuation band,

$$\frac{Z_1}{4Z_2}<-1,\ \text{i.e.}\ \frac{f}{f_c}<1 \qquad f/f_c>1$$

$$\alpha=2\cosh^{-1}\left[\frac{Z_1}{4Z_2}\right]=2\cosh^{-1}\left(\frac{f}{f_c}\right);\ \beta=\pi$$

The plots of α and β for pass and stop bands are shown in Fig.9.10

Thus, from Fig. 9.10, $\alpha = 0$, $\beta = 2 \sinh^{-1} (f/f_C)$ for $f < f_C$

$\alpha = 2\cosh^{-1} (f / f_C)$; $\beta = \pi$ for $f > f_C$

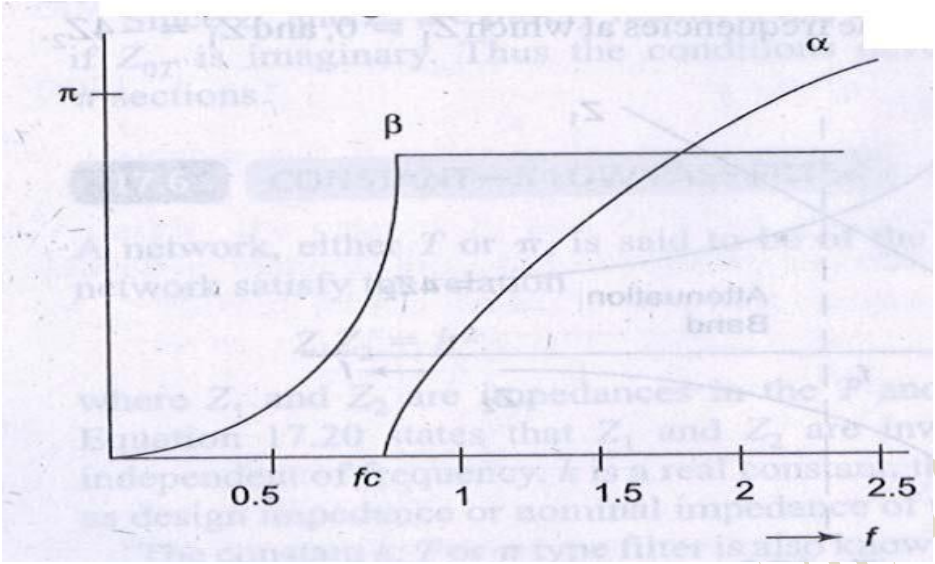


Fig .9.10

The characteristics impedance can be calculated as follows

$$Z_{0T} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

$$= \sqrt{\frac{L}{C}\left(1 - \frac{\omega^2 LC}{4}\right)}$$

$$Z_{0T} = k\sqrt{1 - \left(\frac{f}{f_c}\right)^2}$$

(9.23)

From Eq.9.23, Z_{OT} is rael when $f < f_C$, i.e.in the pass band at $f = f_C$, Z_{OT} ; and for $f > f_C$, Z_{OT} is imaginary in the attenuation band , rising to infinite reactance at infinite frequency . The variation of Z_{OT} with frequency is shown in Fig.9.11

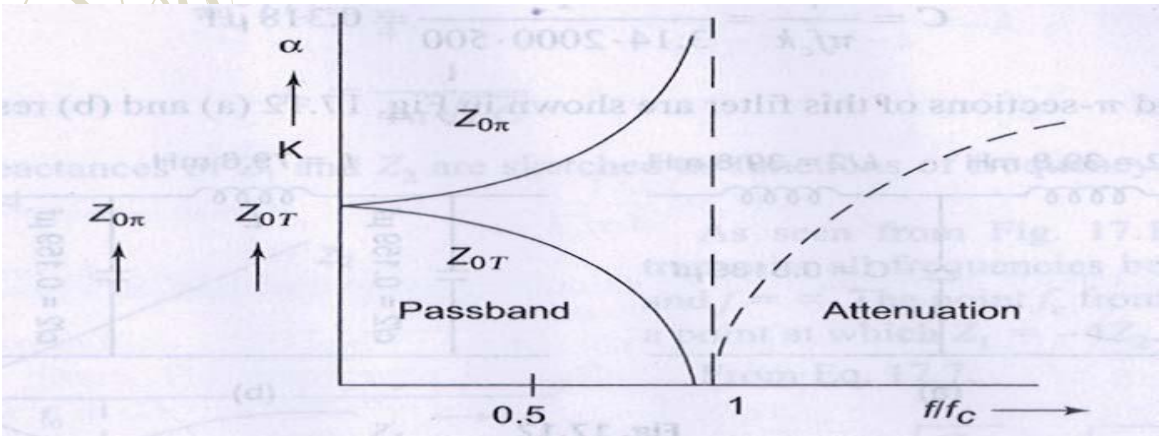


Fig.9.11

Similarly, the characteristics impedance of a π – network is given by

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{k}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}}$$

(9.24)

The variation of $Z_{O\pi}$ with frequency is shown in Fig.9.11 . For $f < f_C$, $Z_{O\pi}$ is real ; at $f = f_C$, Z_{OT} is infinite , and for $f > f_C$, $Z_{O\pi}$ is imaginary . A low pass filter can be designed from the specifications of cut-off frequency and load resistance.

At cut-off frequency, $Z_1 = -4Z_2$

$$j\omega_c L = \frac{-4}{j\omega_c C}$$

$$\pi^2 f_c^2 LC = 1$$

Also we know that $k = \sqrt{L/C}$ is called the design impedance or the load resistance

$$\therefore \quad k^2 = \frac{L}{C}$$

$$\pi^2 f_c^2 k^2 C^2 = 1$$

$C = \frac{1}{\pi f_c k}$ gives the value of the *shunt capacitance*

and $L = k^2 C = \frac{k}{\pi f_c}$ gives the value of the series inductance.

Example 9.1.

Design a low pass filter (both π and T – sections) having a cut-off frequency of 2 *kHz* to operate with a terminated load resistance of 500 Ω .

solution. It is given that $k = \sqrt{(L/C)} = 500\ \Omega$, and $f_C = 2000$ Hz

we know that $L = k/\pi f_C = 500/3.14 \times 2000 = 79.6$ mH

$C = 1/\pi f_C k = 1/3.14.2000.500 = 0.318\ \mu F$

The T and π – sections of this filter are shown in Fig.9.12 (a) and (b) respectively.

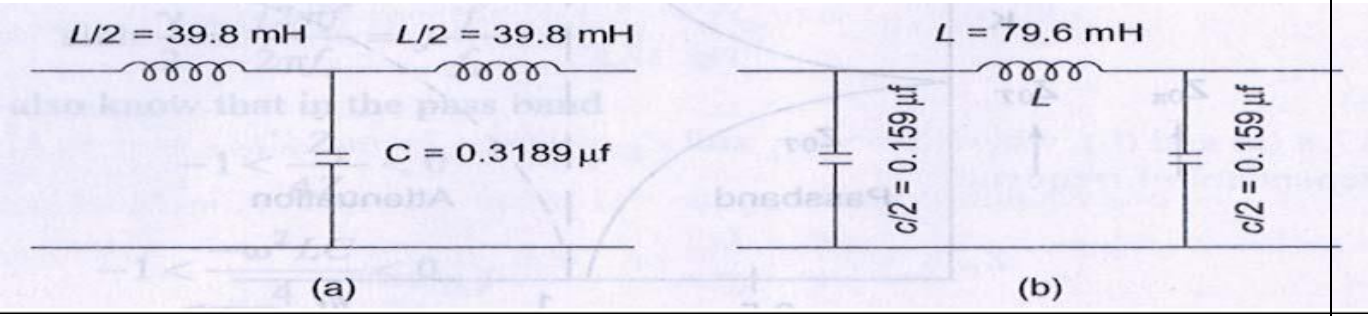


Fig.9.12

9.7 CONSTANT K – HIGH PASS FILTER

Constant K – high pass filter can be obtained by changing the positions of series and shunt arms of the networks shown in Fig.9.8.The prototype high pass filters are shown in Fig.9.13,where $Z_1 = -j/\omega C$ and $Z_2 = j\omega L$.

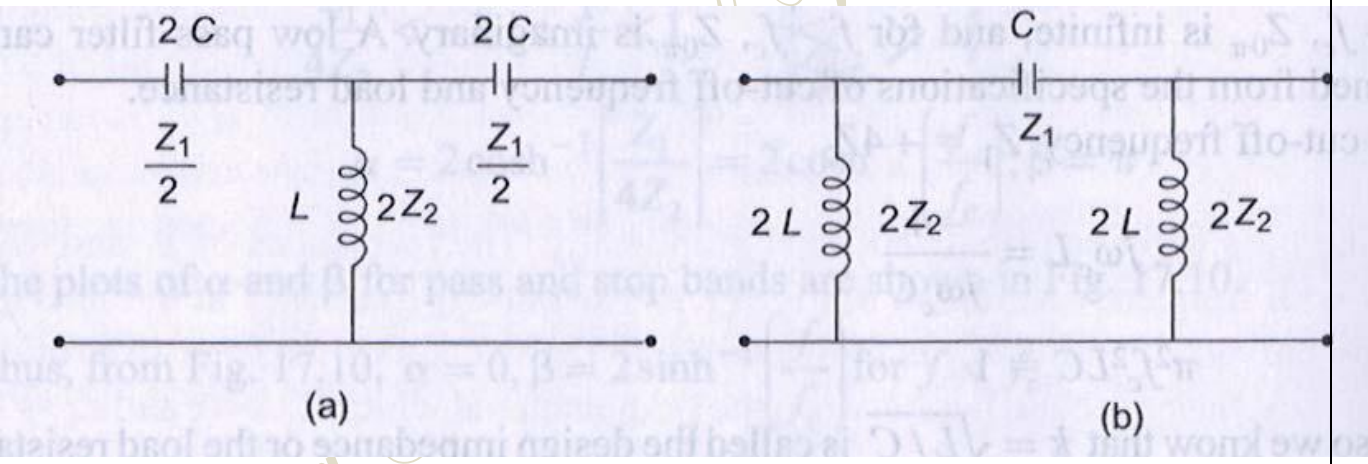


Fig.9.13

Again, it can be observed that the product of Z_1 and Z_2 is independent of frequency, and the filter design obtained will be of the constant k type .Thus, Z_1Z_2 are given by

$$Z_1Z_2 = \frac{-j}{\omega C} j\omega L = \frac{L}{C} = k^2$$

$$k = \sqrt{\frac{L}{C}}$$

The cut-off frequencies are given by $Z_1 = 0$ and $Z_2 = -4Z_2$.

$Z_1 = 0$ indicates $j/\omega C = 0$, or $\omega \rightarrow \alpha$

From $Z_1 = -4Z_2$

$$-j/\omega C = -4\, j\omega L$$

$$\omega^2 LC = 1/4$$

or
$$f_c = \frac{1}{4\pi\sqrt{LC}} \quad (9.25)$$

The reactances of Z_1 and Z_2 are sketched as functions of frequency as shown in Fig.9.14.

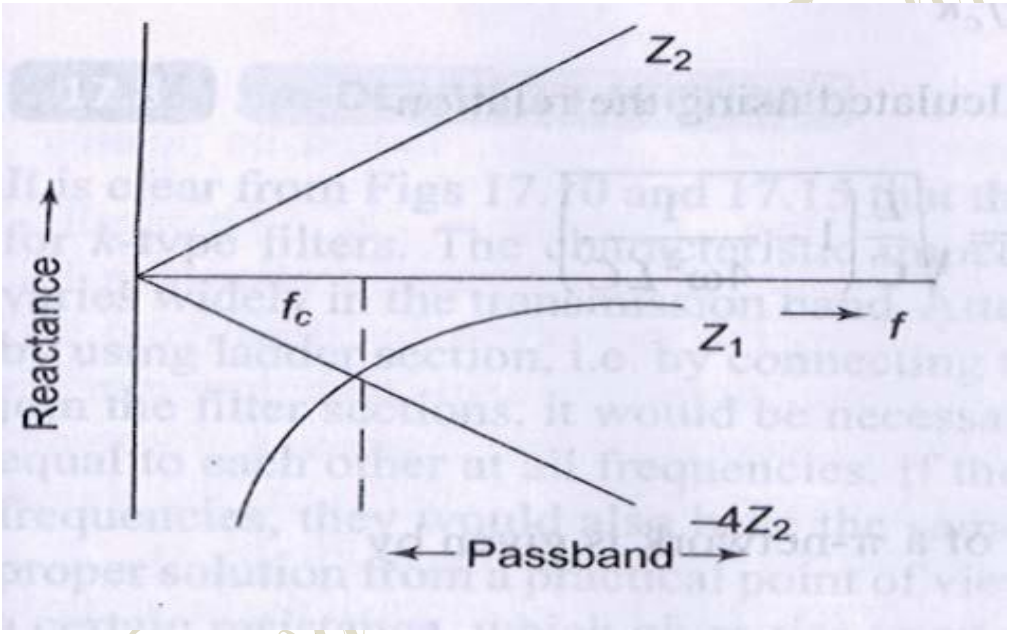


Fig.9.14

As seen from Fig.9.14, the filter transmits all frequencies between $f = f_C$ and $f = \alpha$. The point f_C from the graph is a point at which $Z_1 = -4Z_2$.

From Eq.9.7,

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{-1}{4\omega^2 LC}}$$

From Eq. 9.25,

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

$$\therefore \quad \sqrt{LC} = \frac{1}{4\pi f_c}$$

$$\therefore \quad \sinh\frac{\gamma}{2} = \sqrt{\frac{-(4\pi)^2(f_c)^2}{4\omega^2}} = j\frac{f_c}{f}$$

In the pass band, $-1 < Z_1/4Z_2 < 0$, $\alpha = 0$ or the region in which $f_C / f < 1$ is a pass band $\beta = 2 \sin^{-1} (f_C / f)$

In the attenuation band $Z_1/4Z_2 < -1$,i.e.$f_C / f > 1$

$$\alpha = 2 \cosh^{-1} [Z_1 / 4Z_2]$$

$$= 2 \cos^{-1}(f_C / f) \; ; \; \beta = -\pi$$

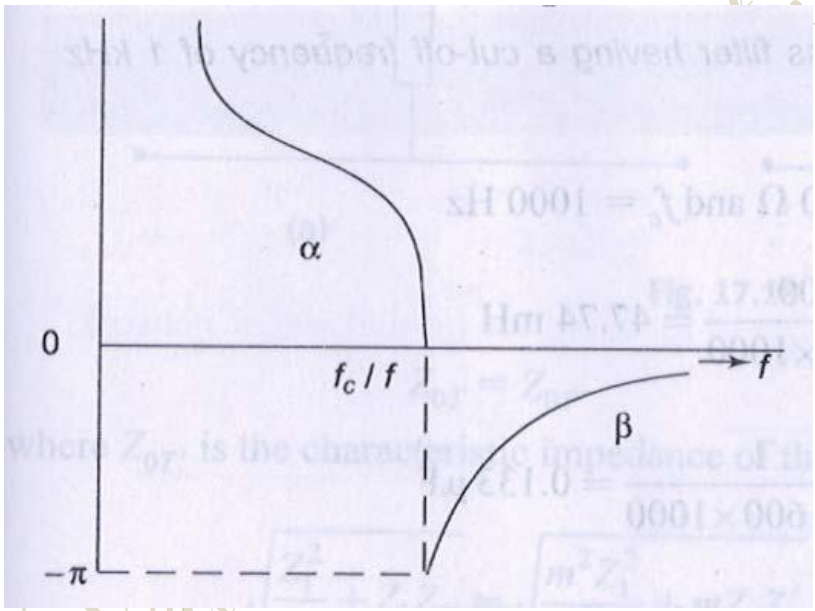


Fig.9.15

The plots of α and β for pass and stop bands of a high pass filter network are shown in Fig.9.15.

A high pass filter may be designed similar to the low pass filter by choosing a resistive load r equal to the constant k , such that $R = k = \sqrt{L/C}$

$$f_c = \frac{1}{4\pi\sqrt{L/C}}$$

$$f_c = \frac{k}{4\pi L} = \frac{1}{4\pi Ck}$$

Since $\sqrt{C} = \frac{L}{k}$,

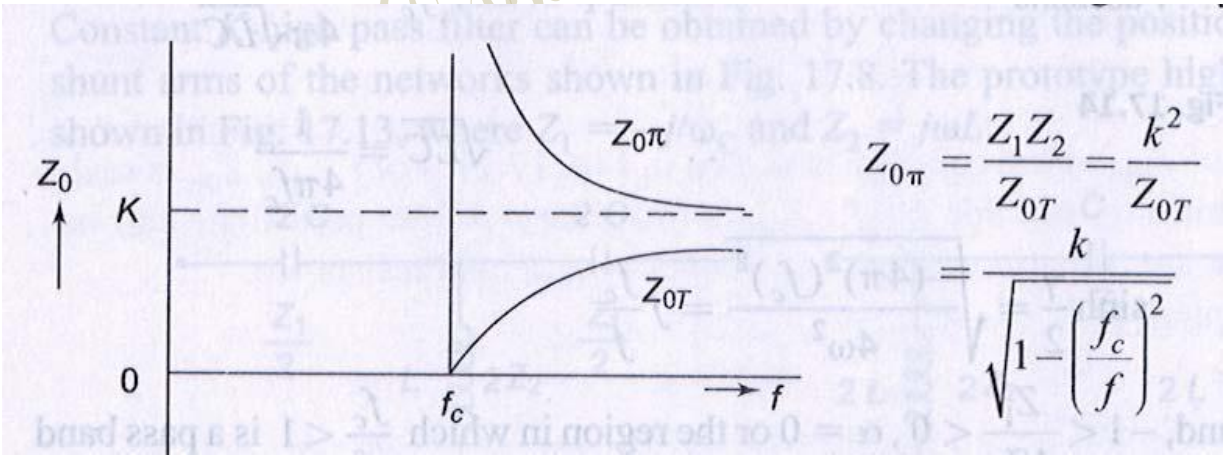
$$L = \frac{k}{4\pi f_c} \text{ and } C = \frac{1}{4\pi f_c k}$$

The characteristic impedance can be calculated using the relation

$$Z_{0T} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)} = \sqrt{\frac{L}{C}\left(1 - \frac{1}{4\omega^2 LC}\right)}$$

$$Z_{0T} = k\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

Similarly, the characteristic impedance of a π – network is given by



$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{k^2}{Z_{0T}} = \frac{k}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

(9.26)

Fig.9.16

The plot of characteristic impedances with respect to frequency is shown in Fig.9.16.

Example 9.2.

Design a high pass filter having a cut-off frequency of 1 kHz with a load resistance of 600 Ω .

Solution. It is given that $R_L = K = 600\ \Omega$ and $f_C = 1000$ Hz

$L = K/4\pi f_c = 600/4 \times \pi \times 1000 = 47.74$ mH

$C = 1/4\pi k f_C = 1/4\pi \times 600 \times 1000 = 0.133\ \mu F$

The T and π – sections of the filter are shown in Fig.9.17.

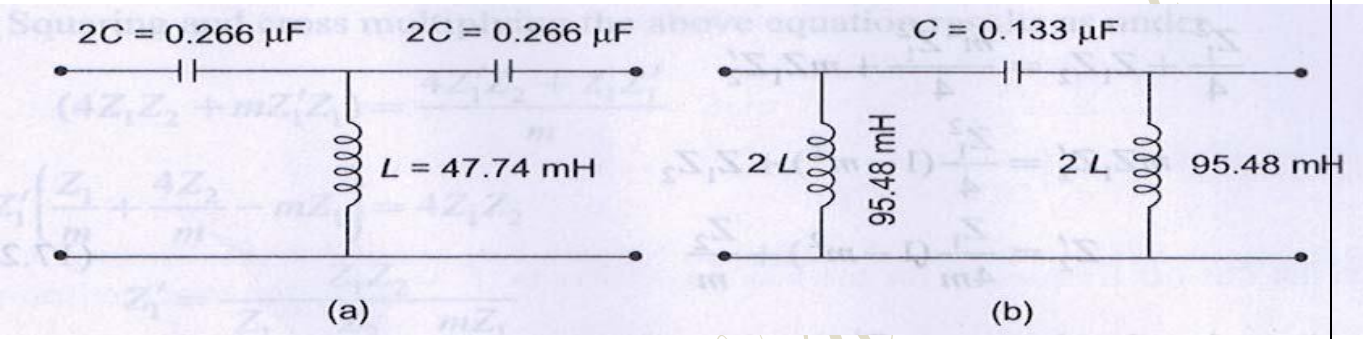


Fig.9.17

9.8 m – DERIVED T – SECTION FILTER

It is clear from Figs.9.10 and 9.15 that the attenuation is not sharp in the stop band for k-type filters. The characteristic impedance, Z_0 is a function of frequency and varies widely in the transmission band. Attenuation can be increased in the stop band by using ladder section, i.e.by connecting two or more identical sections. In order to join the filter sections, it would be necessary that their characteristic impedances be equal to each other at all frequencies. If their characteristic impedances match at all frequencies, they would also have the same pass band . However , cascading is not a proper solution from a practical point of view .

This is because practical elements have a certain resistance, which gives rise to attenuation in the pass band also. Therefore, any attempt to increase attenuation in stop band by cascading also results in an increase of 'α' in the pass band .If the constant k section is regarded as the prototype, it is possible to design a filter to have rapid attenuation in the stop band , and the same characteristic impedance as the prototype at all frequencies . Such a filter is called *m – derived filter*. Suppose a prototype T – network shown in Fig.9.18(a) has the series arm modified as shown in Fig.9.18 (b) , where *m* is a constant . Equating the characteristic impedance of the networks in Fig.9.18, we have

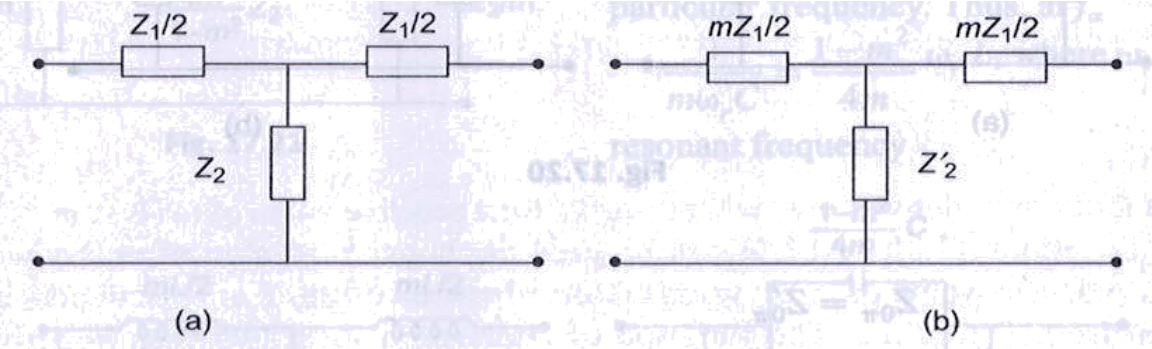


Fig.9.18

$Z_{OT} = Z_{OT}'$

Where Z_{OT} ,is the characteristic impedance of the modified (m – derived) T – network.

$$\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{\frac{m^2 Z_1^2}{4} + m Z_1 Z_2'}$$

$$\frac{Z_1^2}{4} + Z_1 Z_2 = \frac{m^2 Z_1^2}{4} + m Z_1 Z_2'$$

$$m Z_1 Z_2' = \frac{Z_1^2}{4}(1 - m^2) + Z_1 Z_2$$

$$Z_2' = \frac{Z_1}{4m}(1 - m^2) + \frac{Z_2}{m}$$

(9.27)

It appears that the shunt arm Z'_2 consists of two impedances in series as shown in Fig.9.19.

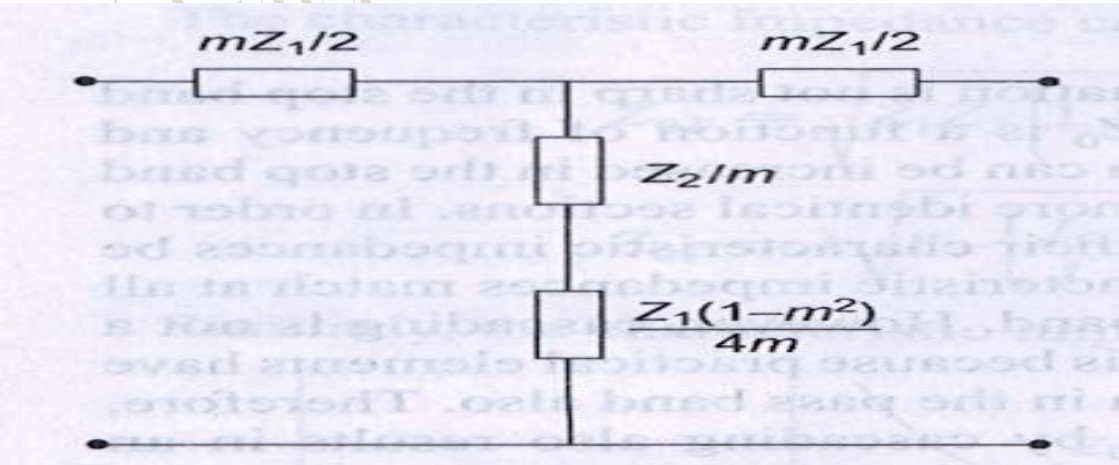


Fig.9.19

From Eq.9.27, $1 - m^2/4m$ should be positive to realize the impedance Z'_2 physically , i.e.0<m<1 . Thus m – derived section can be obtained from the prototype by modifying its series and shunt arms .The same technique can be applied to π section network. Suppose a prototype π – network shown in Fig. 9.20 (a) has the shunt arm modified as shown in Fig. 9.20(b).

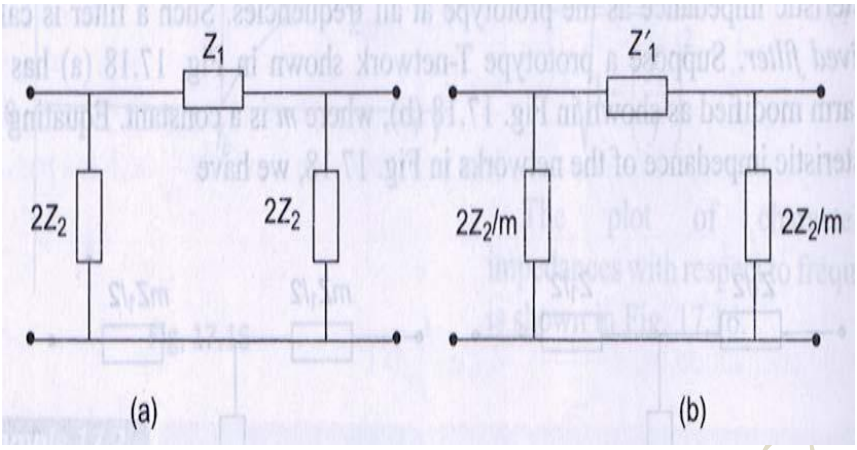


Fig.9.20

$Z_{0\pi} = Z'_{0\pi}$

Where $Z'_{0\pi}$ is the characteristic impedance of the modified (m – derived) π – network.

$$\therefore \quad \sqrt{\frac{Z_1 Z_2}{1+\frac{Z_1}{4Z_2}}} = \sqrt{\frac{Z_1' \frac{Z_2}{m}}{1+\frac{Z_1'}{4 \cdot Z_2 / m}}}$$

Squaring and cross multiplying the above equation results as under.

$$(4Z_1Z_2 + mZ_1'Z_1) = \frac{4Z_1'Z_2 + Z_1Z_1'}{m}$$

$$Z_1'\left(\frac{Z_1}{m} + \frac{4Z_2}{m} - mZ_1\right) = 4Z_1Z_2$$

or
$$Z_1' = \frac{Z_1Z_2}{\frac{Z_1}{4m} + \frac{Z_2}{m} - \frac{mZ_1}{4}}$$

$$= \frac{Z_1Z_2}{\frac{Z_2}{m} + \frac{Z_1}{4m}(1-m^2)}$$

$$Z_1' = \frac{Z_1Z_2\frac{4m^2}{(1-m^2)}}{\frac{Z_2 4m^2}{m(1-m^2)} + Z_1m} = \frac{mZ_1\frac{Z_2 4m}{(1-m^2)}}{mZ_1 + \frac{Z_2 4m}{(1-m^2)}}$$

(9.28)

It appears that the series arm of the m – derived π section is a parallel combination of mZ_1 and $4mZ_2 /1 - m^2$. The derived m section is shown in Fig.9.21.

m – Derived Low Pass Filter

In Fig.9.22 , both *m* – derived low pass T and π filter sections are shown. For the T –section shown in Fig.9.22(a) , the shunt arm is to be chosen so that it is resonant at some frequency f_α above cut-off frequency fc .

If the shunt arm is series resonant ,its impedance will be minimum or zero .Therefore , the output is zero and will correspond to infinite attenuation at this particular frequency . Thus, at f_α

$1/m\omega_r C = 1 - m^2/4m\ \omega_r L$, where ω_r is the resonant frequency

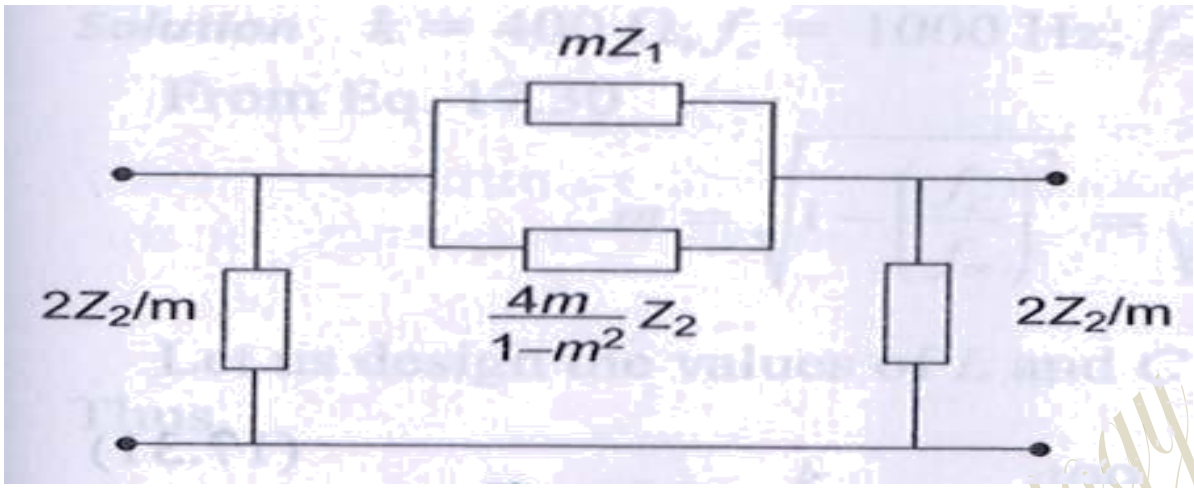


Fig.9.21

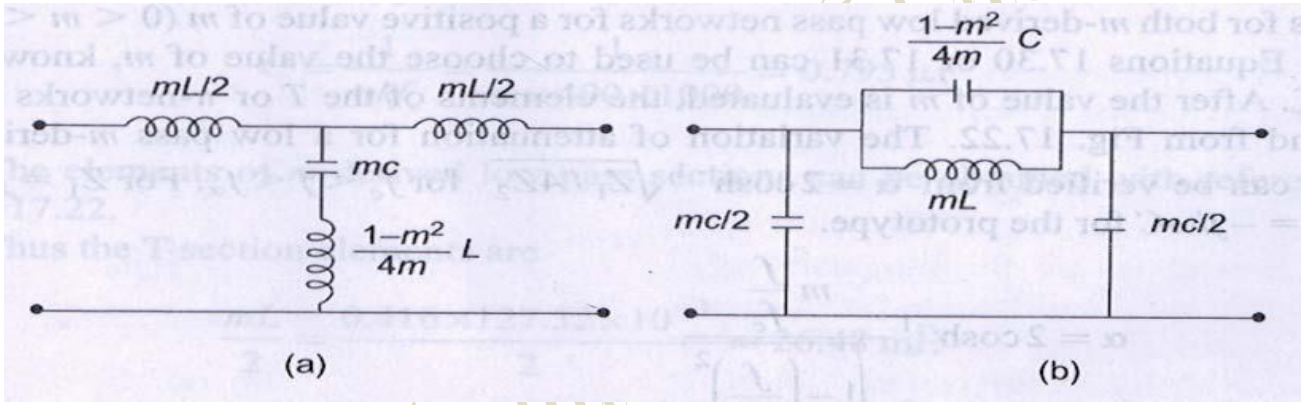


Fig.9.22

$$\omega_r^2 = \frac{4}{(1-m^2)LC}$$

$$f_r = \frac{1}{\pi\sqrt{LC(1-m^2)}} = f_\infty$$

Since the cut-off frequency for the low pass filter is $f_c = 1/\pi\sqrt{LC}$

$$f_\infty = \frac{f_c}{\sqrt{1-m^2}}$$

(9.29)

or $$m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2}$$

(9.30)

If a sharp cut-off is desired,f_α should be near to f_c . From Eq.9.29,it is clear that for the smaller the value of m,f_α comes close to f_c .Equation 9.30 shows that if f_c and f_α are specified , the necessary value of m may then be calculated. Similarly, for m – derived π section, the inductance and capacitance in the series arm constitute a resonant circuit . Thus , at f_α a frequency corresponds to infinite attenuation, i.e. at f_α

$$m\omega_r L = \frac{1}{\left(\frac{1-m^2}{4m}\right)\omega_r C}$$

$$\omega_r^2 = \frac{4}{LC(1-m^2)}$$

$$f_r = \frac{1}{\pi\sqrt{LC(1-m^2)}}$$

Since, $f_c = \frac{1}{\pi\sqrt{LC}}$

$$f_r = \frac{f_c}{\sqrt{1-m^2}} = f_\alpha$$

(9.31)

Thus for both m – derived low pass networks for a positive value of $m(0 < m < 1)$, $f_\alpha > f_c$. Equations 9.30 or 9.31 can be used to choose the value of m, knowing f_c and f_r . After the value of m is evaluated, the elements of the T or π – networks can be found from Fig.9.22. The variation of attenuation for a low pass m – derived section can be verified from $\alpha = 2\cosh^{-1}\sqrt{Z_1/4Z_2}$ for $f_c < f < f_\alpha$. For $Z_1 = j\omega L$ and $Z_2 = -j/\omega C$ for the prototype.

$$\therefore \quad \alpha = 2\cosh^{-1}\frac{m\frac{f}{f_c}}{\sqrt{1-\left(\frac{f}{f_\alpha}\right)^2}}$$

and $$\beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_1}\right|} = 2\sin^{-1}\frac{m\frac{f}{f_c}}{\sqrt{1-\left(\frac{f}{f_c}\right)^2(1-m)^2}}$$

Figure 9.23 shows the variation of α , β and Z_0 with respect to frequency for an m – derived low pass filter.

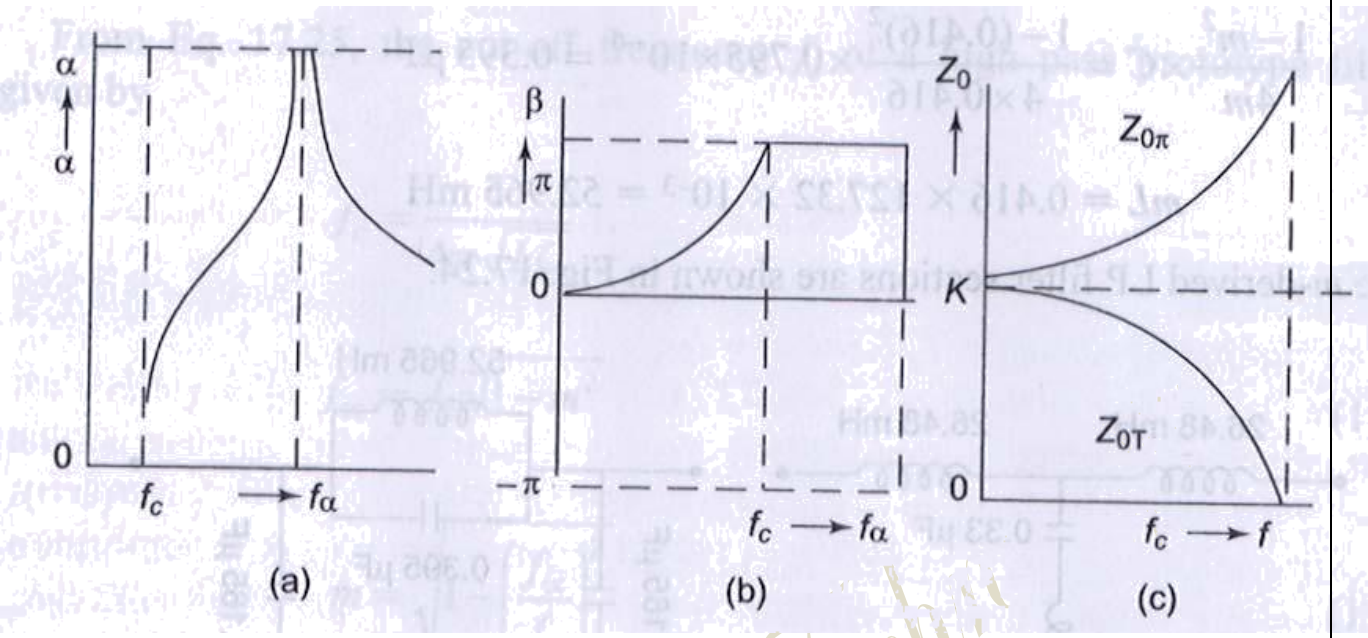


Fig.9.23

Example 9.3

Design a m – derived low pass filter having cut-off frequency of 1kHz, design impedance of 400 Ω, and the resonant frequency 1100 Hz.

Solution. k = 400 Ω, f_C =1000 Hz ; f_α =1100 Hz

From Eq.9.30

$$m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2} = \sqrt{1-\left(\frac{1000}{1100}\right)^2} = 0.416$$

Let us design the values of L and C for a low pass , K – type filter (prototype filter).

Thus,

$$L = \frac{k}{\pi f_c} = \frac{400}{\pi \times 1000} = 127.32 \text{ mH}$$

$$C = \frac{1}{\pi k f_c} = \frac{1}{\pi \times 400 \times 1000} = 0.795 \ \mu\text{F}$$

The elements of *m* – derived low pass sections can be obtained with reference to Fig.9.22.

Thus the T-section elements are

$$\frac{mL}{2} = \frac{0.416 \times 127.32 \times 10^{-3}}{2} = 26.48 \text{ mH}$$

$$mC = 0.416 \times 0.795 \times 10^{-6} = 0.33 \ \mu\text{F}$$

$$\frac{1-m^2}{4m} L = \frac{1-(0.416)^2}{4 \cdot 0.416} \times 127.32 \times 10^{-3} = 63.27 \text{ mH}$$

The π-section elements are

$$\frac{mC}{2} = \frac{0.416 \times 0.795 \times 10^{-6}}{2} = 0.165 \ \mu\text{F}$$

$$\frac{1-m^2}{4m} \times C = \frac{1-(0.416)^2}{4 \times 0.416} \times 0.795 \times 10^{-6} = 0.395 \ \mu\text{F}$$

$$mL = 0.416 \times 127.32 \times 10^{-3} = 52.965 \text{ mH}$$

The *m* –derived LP filter sections are shown in Fig.9.24.

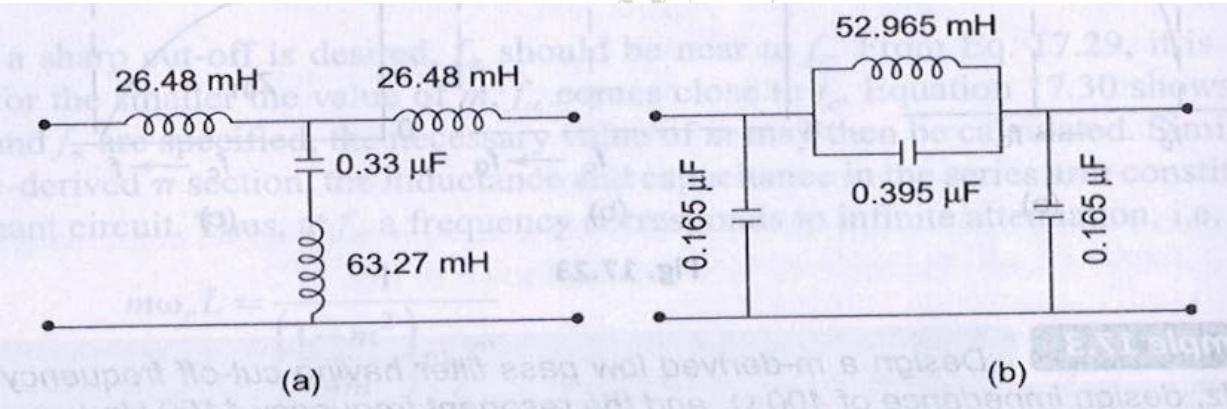


Fig.9.24

***m* – Derived High Pass Filter**

In Fig.9.25 both *m* – derived high pass T and π – section are shown.

If the shunt arm in T – section is series resonant, it offers minimum or zero impedance.Therefore, the output is zero and, thus, at resonance frequency or the frequency corresponds to infinite attenuation.

$$\omega_r \frac{L}{m} = \frac{1}{\omega_r \frac{4m}{1-m^2} C}$$

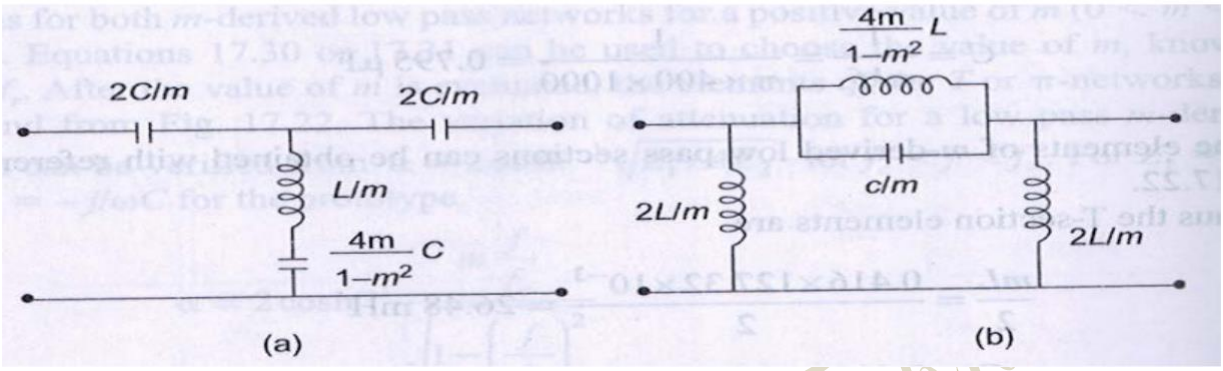


Fig.9.25

$$\omega_r^2 = \omega_\infty^2 = \frac{1}{\frac{L}{m}\frac{4m}{1-m^2} C} = \frac{1-m^2}{4LC}$$

$$\omega_\infty = \frac{\sqrt{1-m^2}}{2\sqrt{LC}} \text{ or } f_\infty = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$

From Eq. 9.25, the cut – off frequency f_C of a high pass prototype filter is given by

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

$$f_\infty = f_c\sqrt{1-m^2}$$

(9.32)

$$m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2}$$

(9.33)

Similarly,for the *m* – derived π – section , the resonant circuit is constituted by the series arm inductance and capacitance . Thus , at f_α

$$\frac{4m}{1-m^2}\omega_r L = \frac{1}{\frac{\omega_r}{m}C}$$

$$\omega_r^2 = \omega_\infty^2 = \frac{1-m^2}{4LC}$$

$$\omega_\infty = \frac{\sqrt{1-m^2}}{2\sqrt{LC}} \quad \text{or} \quad f_\infty = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$

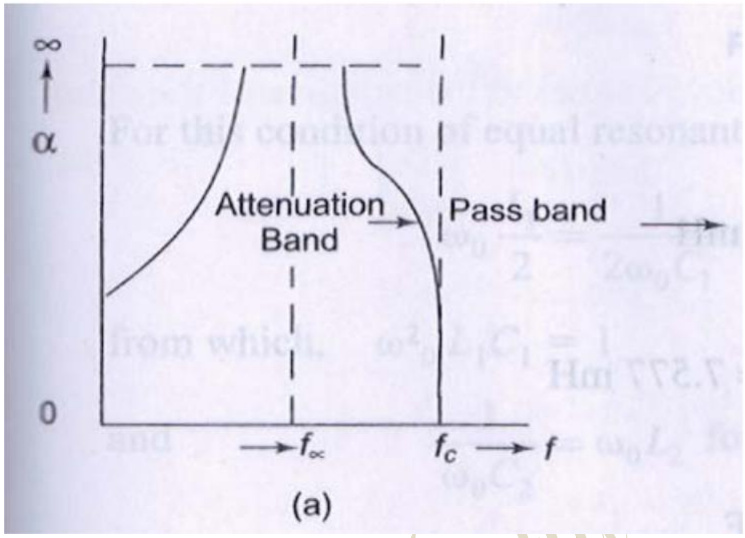


Fig.9.26

Thus the frequency corresponding to infinite attenuation is the same for both sections. Equation 9.33 may be used to determine *m* for a given f_α and f_C . The elements of the *m* – derived high pass *T* or π – sections can be found from Fig.9.25. The variation of α, β and Z_0 with frequency is shown in Fig. 9.26.

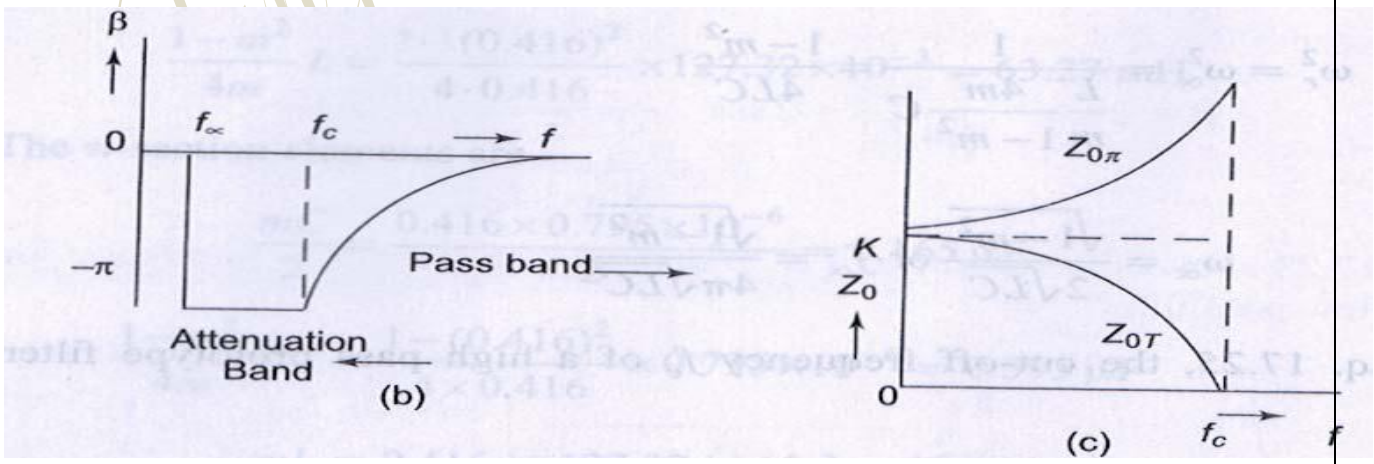


Fig.9.26

Example 9.4.

Design a m-derived high pass filter with a cut-off frequency of 10kHz; design impedance of 5Ω and m = 0.4.

Solution . For the prototype high pass filter,

$$L = \frac{k}{4\pi f_c} = \frac{500}{4 \times \pi \times 10000} = 3.978 \text{ mH}$$

$$C = \frac{1}{4\pi k f_c} = \frac{1}{4\pi \times 500 \times 10000} = 0.0159 \ \mu\text{F}$$

The elements of m-derived high pass sections can be obtained with reference to Fig.9.25.Thus, the T-section elements are

$$\frac{2C}{m} = \frac{2 \times 0.0159 \times 10^{-6}}{0.4} = 0.0795 \ \mu\text{F}$$

$$\frac{L}{m} = \frac{3.978 \times 10^{-3}}{0.4} = 9.945 \text{ mH}$$

$$\frac{4m}{1-m^2} C = \frac{4 \times 0.4}{1-(0.4)^2} \times 0.0159 \times 10^{-6} = 0.0302 \ \mu\text{F}$$

The π-section elements are

$$\frac{2L}{m} = \frac{2 \times 0.0159 \times 10^{-3}}{0.4} = 19.89 \text{ mH}$$

$$\frac{4m}{1-m^2} \times L = \frac{4 \times 0.4}{1-(0.4)^2} \times 3.978 \times 10^{-3} = 7.577 \text{ mH}$$

$$\frac{C}{m} = \frac{0.0159}{0.4} \times 10^{-6} = 0.0397 \ \mu\text{F}$$

T and π sections of the m –derived high pass filter are shown in Fig.9.27.

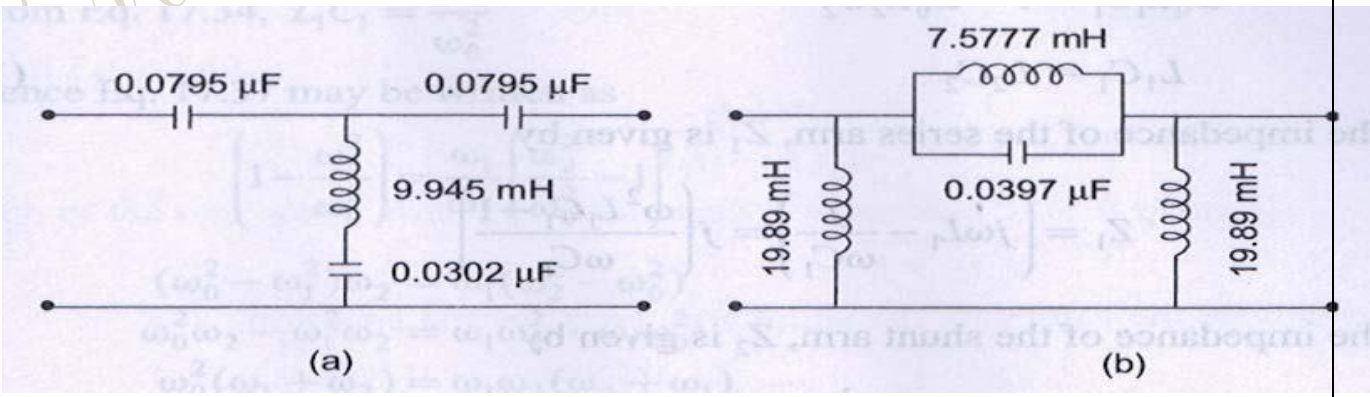


Fig.9.27

9.9 BAND PASS FILTER

As already explained in Section 9.1 , a band pass filter is one which attenuates all frequencies below a lower cut-off frequency f_1 and above an upper cut-off frequency f_2 . Frequencies lying between f_1 and f_2 comprise the pass band ,and are transmitted with zero attenuation .A band pass filter may be obtained by using a low pass filter followed by a high pass filter in which the cut-off frequency of the LP filter is above the cut-off frequency of the HP filter , the overlap thus allowing only a band of frequencies to pass . This is not economical in practice; it is more economical to combine the low and high pass functions into a single filter section .

Consider the circuit in Fig.9.28, each arm has a resonant circuit with same resonant frequency, i.e. the resonant frequency of the series arm and the resonant frequency of the shunt arm are made equal to obtain the band pass characteristic.

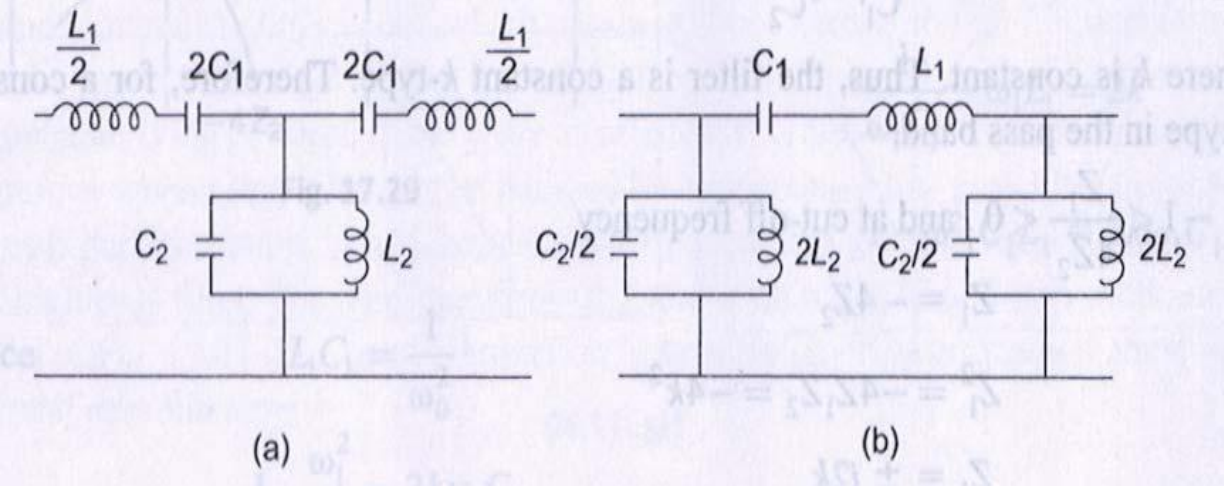


Fig.9.28

For this condition of equal resonant frequencies.

$$\omega_0 \frac{L_1}{2} = \frac{1}{2\omega_0 C_1} \quad \text{for the series arm}$$

from which, $\omega^2_0 L_1 C_1 = 1$

(9.34)

and $\dfrac{1}{\omega_0 C_2} = \omega_0 L_2$ for the shunt arm

from which, $\omega_0^2 L_2 C_2 = 1$

(9.35)

$$\omega_0^2 L_1 C_1 = 1 = \omega_0^2 L_2 C_2$$
$$L_1 C_1 = L_2 C_2$$

(9.36)

The impedance of the series arm, Z_1 is given by

$$Z_1 = \left(j\omega L_1 - \frac{j}{\omega C_1} \right) = j\left(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1} \right)$$

The impedance of the shunt arm, Z_2 is given by

$$Z_2 = \frac{j\omega L_2 \dfrac{1}{j\omega C_2}}{j\omega L_2 + \dfrac{1}{j\omega C_2}} = \frac{j\omega L_2}{1 - \omega^2 L_2 C_2}$$

$$Z_1 Z_2 = j\left(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1} \right)\left(\frac{j\omega L_2}{1 - \omega^2 L_2 C_2} \right)$$
$$= \frac{-L_2}{C_1}\left(\frac{\omega^2 L_1 C_1 - 1}{1 - \omega^2 L_2 C_2} \right)$$

From Eq.9.36

$$L_1 C_1 = L_2 C_2$$
$$Z_1 Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2} = k^2$$

Where *k* is constant. Thus, the filter is a constant *k* – type .Therefore, for a constant *k* – type in the pass band.

$$-1 < \frac{Z_1}{4Z_2} < 0$$, and at cut-off frequency
$$Z_1 = -4Z_2$$
$$Z_1^2 = -4Z_1 Z_2 = -4k^2$$
$$\therefore \quad Z_1 = \pm j2k$$

i.e. the value of Z_1 at lower cut-off frequency is equal to the negative of the value of Z_1 at the upper cut-off frequency .

$$\therefore \quad \left(\frac{1}{j\omega_1 C_1} + j\omega_1 L_1\right) = -\left(\frac{1}{j\omega_2 C_1} + j\omega_2 L_1\right)$$

$$\text{or} \quad \left(\omega_1 L_1 - \frac{1}{\omega_1 C_1}\right) = \left(\frac{1}{\omega_2 C_1} - \omega_2 L_1\right)$$

$$(1 - \omega_1^2 L_1 C_1) = \frac{\omega_1}{\omega_2}(\omega_2^2 L_1 C_1 - 1)$$

(9.37)

From Eq.9.34, $L_1C_1 = 1/\omega_0^2$

Hence Eq.9.37 may be written as

$$\left(1 - \frac{\omega_1^2}{\omega_0^2}\right) = \frac{\omega_1}{\omega_2}\left(\frac{\omega_2^2}{\omega_0^2} - 1\right)$$

$$(\omega_0^2 - \omega_1^2)\omega_2 = \omega_1(\omega_2^2 - \omega_0^2)$$

$$\omega_0^2\omega_2 - \omega_1^2\omega_2 = \omega_1\omega_2^2 - \omega_1\omega_0^2$$

$$\omega_0^2(\omega_1 + \omega_2) = \omega_1\omega_2(\omega_2 + \omega_1)$$

$$\omega_0^2 = \omega_1\omega_2$$

$$f_0 = \sqrt{f_1 f_2}$$

(9.38)

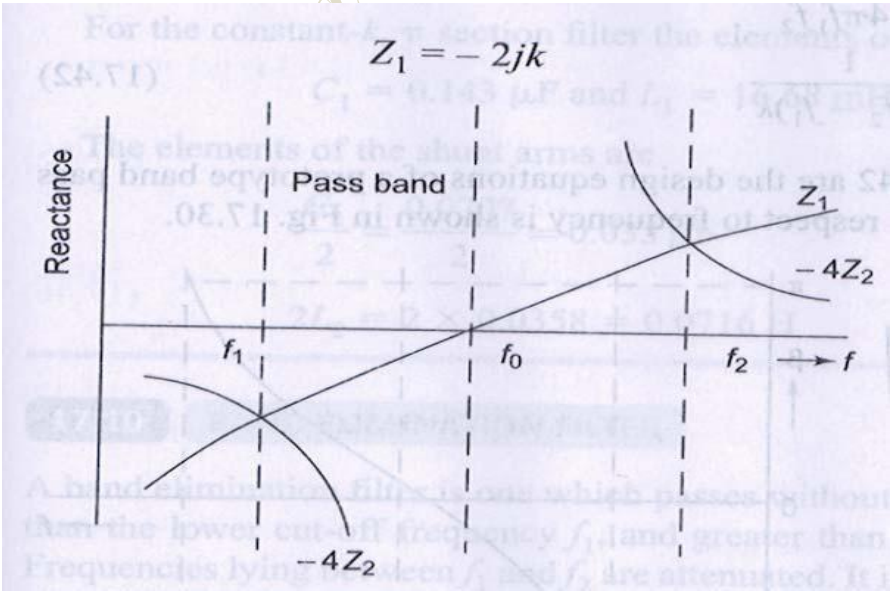


Fig.9.29

Thus, the resonant frequency is the geometric mean of the cut-off frequencies. The variation of the reactances with respect to frequency is shown in Fig.9.29.

If the filter is terminated in a load resistance $R = K$, then at the lower cut-off frequency.

$$\left(\frac{1}{j\omega_1 C_1} + j\omega_1 L_1\right) = -2jk$$

$$\frac{1}{\omega_1 C_1} - \omega_1 L_1 = 2k$$

$$1 - \omega^2{}_1 C_1 L_1 = 2k\omega_1 C_1$$

Since $L_1 C_1 = \frac{1}{\omega_0^2}$

$$1 - \frac{\omega_1^2}{\omega_0^2} = 2k\omega_1 C_1$$

or $$1 - \left(\frac{f_1}{f_0}\right)^2 = 4\pi k f_1 C_1$$

$$1 - \frac{f_1^2}{f_1 f_2} = 4\pi k f_1 C_1 \qquad (\because f_0 = \sqrt{f_1 f_2})$$

$$f_2 - f_1 = 4\pi k f_1 f_2 C_1$$

$$C_1 = \frac{f_2 - f_1}{4\pi k f_1 f_2}$$

(9.39)

Since $L_1 C_1 = \frac{1}{\omega_0^2}$

$$L_1 = \frac{1}{\omega_0^2 C_1} = \frac{4\pi k f_1 f_2}{\omega_0^2 (f_2 - f_1)}$$

$$L_1 = \frac{k}{\pi (f_2 - f_1)}$$

(9.40)

To evaluate the values for the shunt arm, consider the equation

$$Z_1 Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2} = k^2$$

$$\therefore \quad L_2 = C_1 k^2 = \frac{(f_2 - f_1)k}{4\pi f_1 f_2}$$

(9.41)

and $$C_2 = \frac{L_1}{k^2} = \frac{1}{\pi(f_2 - f_1)k}$$

(9.42)

Equations 9.39 through 9.42 are the design equations of a prototype band pass filter. T he variation of α , β with respect to frequency is shown in Fig.9.30 .

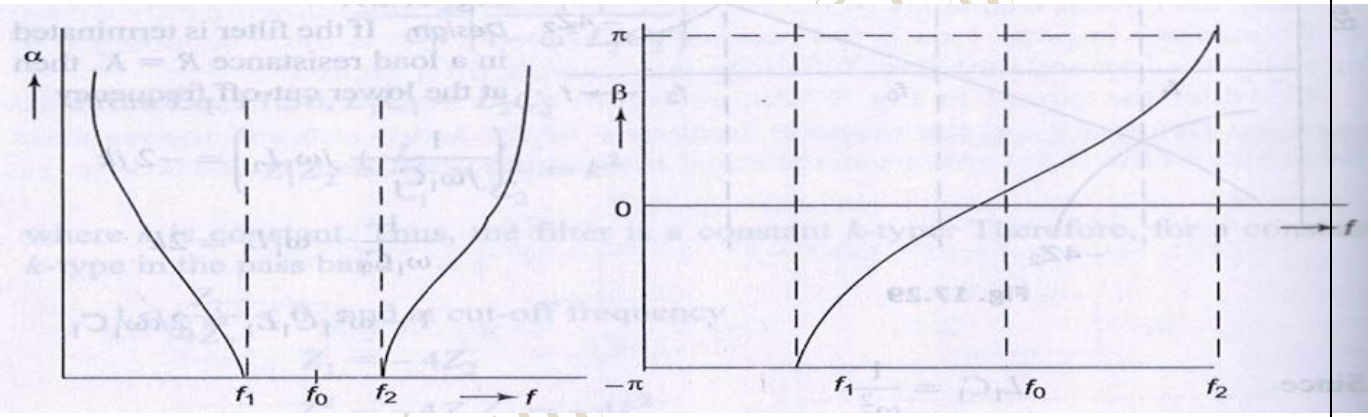


Fig.9.30

Example 9.5.

Design k – type band pass filter having a design impedance of 500 Ω and cut-off frequencies 1 kHz and 10 kHz.

Solution .

k = 500 Ω; f_1 = 1000 Hz; f_2 = 10000

Hz From Eq.9.40,

$$L_1 = \frac{k}{\pi(f_2 - f_1)} = \frac{500}{\pi 9000} = \frac{55.55}{\pi}\text{mH} = 16.68\text{ mH}$$

From Eq.9.39,

$$C_1 = \frac{f_2 - f_1}{4\pi k f_1 f_2} = \frac{9000}{4 \times \pi \times 500 \times 1000 \times 10000} = 0.143\,\mu\text{F}$$

From Eq.9.41,

$$L_2 = C_1 k^2 = 3.57 \text{ mH}$$

From Eq.9.42,

$$C_2 = \frac{L_1}{k^2} = 0.0707 \,\mu\text{F}$$

Each of the two series arms of the constant *k*, *T* – section filter is given by

$$\frac{L_1}{2} = \frac{17.68}{2} = 8.84 \text{ mH}$$

$$2C_1 = 2 \times 0.143 = 0.286 \,\mu\text{F}$$

And the shunt arm elements of the network are given by

$$C_2 = 0.0707 \,\mu\text{F and } L_2 = 3.57 \text{ mH}$$

For the constant-*k*, π section filter the elements of the series arm are

$$C_1 = 0.143 \,\mu\text{F and } L_1 = 16.68 \text{ mH}$$

The elements of the shunt arms are

$$\frac{C_2}{2} = \frac{0.0707}{2} = 0.035 \,\mu\text{F}$$

$$2L_2 = 2 \times 0.0358 = 0.0716 \text{ H}$$

9.10 BAND ELIMINATION FILTER

A band elimination filter is one which passes without attenuation all frequencies less than the lower cut-off frequency f_1 , and greater than the upper cut-off frequency f_2 . Frequencies lying between f_1 and f_2 are attenuated. It is also known as band stop filter. Therefore, a band stop filter can be realized by connecting a low pass filter in parallel with a high pass section, in which the cut-off frequency of low pass filter is below that of a high pass filter. The configurations of *T* and π constant *k* band stop sections are shown in Fig.9.31. The band elimination filter is designed in the same manner as is the band pass filter.

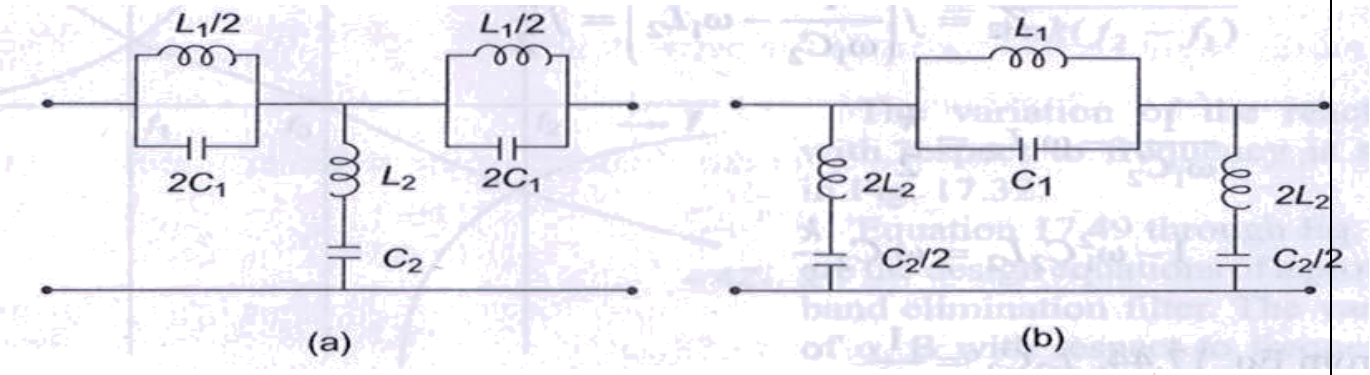


Fig.9.31

As for the band pass filter, the series and shunt arms are chosen to resonate at the same frequency ω_0. Therefore, from Fig.9.31 (a), for the condition of equal resonant frequencies

$$\frac{\omega_0 L_1}{2} = \frac{1}{2\omega_0 C_1} \quad \text{for the series arm}$$

or

$$\omega_0^2 = \frac{1}{L_1 C_1} \tag{9.43}$$

$$\omega_0 L_2 = \frac{1}{\omega_0 C_2} \quad \text{for the shunt arm}$$

$$\omega_0^2 = \frac{1}{L_2 C_2} \tag{9.44}$$

$$\frac{1}{L_1 C_1} = \frac{1}{L_2 C_2} = k$$

Thus

$$L_1 C_1 = L_2 C_2 \tag{9.45}$$

It can be also verified that

$$Z_1 Z_2 = \frac{L_1}{C_2} = \frac{L_2}{C_1} = k^2 \tag{9.46}$$

and

$$f_0 = \sqrt{f_1 f_2} \tag{9.47}$$

At cut-off frequencies, $Z_1 = -4Z_2$

Multiplying both sides with Z_2, we get

$$Z_1 Z_2 = -4Z_2^2 = k^2$$
$$Z_2 = \pm j\frac{k}{2}$$

(9.48)

If the load is terminated in a load resistance, $R = k$, then at lower cut-off frequency

$$Z_2 = j\left(\frac{1}{\omega_1 C_2} - \omega_1 L_2\right) = j\frac{k}{2}$$
$$\frac{1}{\omega_1 C_2} - \omega_1 L_2 = \frac{k}{2}$$
$$1 - \omega_1^2 C_2 L_2 = \omega_1 C_2 \frac{k}{2}$$

From Eq.9.44,

$$L_2 C_2 = \frac{1}{\omega_0^2}$$

$$1 - \frac{\omega_1^2}{\omega_0^2} = \frac{k}{2}\omega_1 C_2$$
$$1 - \left(\frac{f_1}{f_0}\right)^2 = k\pi f_1 C_2$$
$$C_2 = \frac{1}{k\pi f_1}\left[1 - \left(\frac{f_1}{f_0}\right)^2\right]$$

Since $f_0 = \sqrt{f_1 f_2}$

$$C_2 = \frac{1}{k\pi}\left[\frac{1}{f_1} - \frac{1}{f_2}\right]$$
$$C_2 = \frac{1}{k\pi}\left[\frac{f_2 - f_1}{f_1 f_2}\right]$$

(9.49)

From Eq.9.44,

$$\omega_0^2 = \frac{1}{L_2 C_2}$$
$$L_2 = \frac{1}{\omega_0^2 C_2} = \frac{\pi k f_1 f_2}{\omega_0^2 (f_2 - f_1)}$$

Since $f_0 = \sqrt{f_1 f_2}$

$$L_2 = \frac{k}{4\pi(f_2 - f_1)}$$

(9.50)

Also from Eq. 9.46,

$$k^2 = \frac{L_1}{C_2} = \frac{L_2}{C_1}$$

$$L_1 = k^2 C_2 = \frac{k}{\pi}\left(\frac{f_2 - f_1}{f_1 f_2}\right)$$

(9.51)

$$\text{and } C_1 = \frac{L_2}{k^2}$$

(9.52)

$$= \frac{1}{4\pi k(f_2 - f_1)}$$

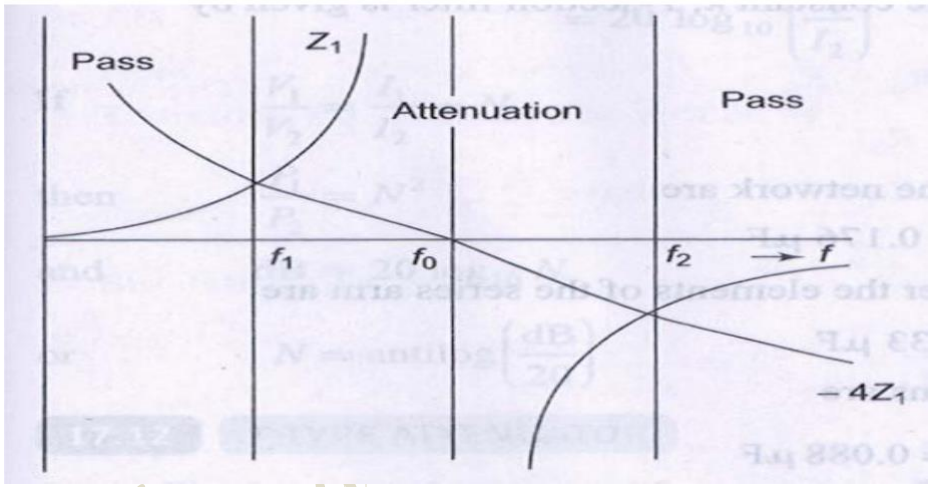


Fig.9.32

The variation of reactances with respect to frequency is shown in Fig.9.32. Equation 9.49 through Eq.9.52 is the design equations of a prototype band elimination filter. The variation of α ,β with respect to frequency is shown in Fig.9.33 .

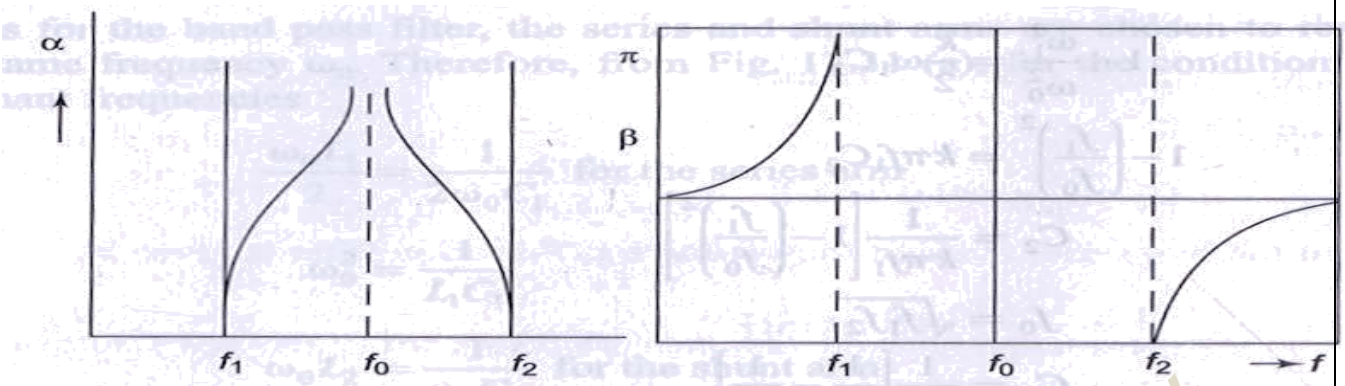


Fig.9.33

Example 9.6.

Design a band elimination filter having a design impedance of 600 Ω and cut-off frequencies f_1 = 2 kHz and f_2 = 6 kHz.

Solution. $(f_2 - f_1)$ = 4 kHz

Making use of the Eqs.9.49 through 9.52 in Section 9.10, we have

$$L_1 = \frac{k}{\pi}\left(\frac{f_2 - f_1}{f_2 f_1}\right) = \frac{600 \times 4000}{\pi \times 2000 \times 6000} = 63 \text{ mH}$$

$$C_1 = \frac{1}{4\pi k(f_2 - f_1)} = \frac{1}{4 \times \pi \times 600(4000)} = 0.033 \ \mu\text{F}$$

$$L_2 = \frac{1}{4\pi k(f_2 - f_1)} = \frac{600}{4\pi(4000)} = 12 \text{ mH}$$

$$C_2 = \frac{1}{k\pi}\left[\frac{f_2 - f_1}{f_1 f_2}\right] = \frac{1}{600 \times \pi}\left[\frac{4000}{2000 \times 6000}\right] = 0.176 \ \mu\text{F}$$

Each of the two series arms of the constant k, T-section filter is given by

$$\frac{L_1}{2} = 31.5 \text{ mH}$$

$$2C_1 = 0.066 \ \mu\text{F}$$

And the shunt arm elements of the network are

$$L_2 = 12 \text{ mH and } C_2 = 0.176 \ \mu\text{F}$$

For the constant k, π-section filter the elements of the series arm are

$$L_1 = 63 \text{ mH}, C_1 = 0.033 \ \mu\text{F}$$